Chapter 1

The Advanced Mathematics Curriculum in the Participating Countries

The mathematics assessment for TIMSS Advanced 2008 was developed according to a framework designed to reflect the mathematics studied around the world in advanced mathematics programs during the final year of schooling. More specifically, the TIMSS Advanced 2008 mathematics framework[1] was organized around content domains and cognitive domains. The content domains or subject matter to be assessed included algebra, calculus, and geometry, while the cognitive domains or thinking behaviors expected of students as they engaged with the mathematics content included knowing, applying, and reasoning. The TIMSS Advanced 2008 countries participated in the iterative review process used to develop the framework and worked collaboratively with the TIMSS & PIRLS International Study Center to develop the test questions (items) covering the framework. Although all countries agreed that the mathematics described in the framework and addressed by the items in the assessment represented a reasonable fit to their curricular goals, it must be emphasized that each of the 10 participating countries had its own approach to teaching and learning advanced mathematics. To better understand the results, therefore, it is important first to understand the differences in the education systems

1 Garden, R.A., Lie, S., Robitaille, D.F., Angell, C., Martin, M.O., Mullis, I.V.S., Foy, P., and Arora, A. (2006). *TIMSS Advanced 2008 assessment frameworks*. Chestnut Hill, MA: TIMSS & PIRLS International Study Center, Boston College.

in the participating countries and the characteristics of the students assessed for TIMSS Advanced.

Because the participating countries had substantive differences in their approaches to educating students in advanced mathematics, the first section of Chapter 1 contains information about the structure of the educational systems in the countries that participated in TIMSS Advanced 2008, with a particular focus on the number of years of schooling involved and the selectivity of the program or track assessed by TIMSS Advanced. Data are presented about the characteristics of the advanced mathematics curriculum in each country, and about the students who participated. Later sections deal with the amount of instructional time allocated to mathematics in these advanced programs or tracks, the degree to which certain topics from the TIMSS Advanced mathematics framework were taught, and the extent to which teachers indicated that they felt well-qualified to teach advanced mathematics.

In comparing achievement across countries, it is important to consider differences in students' curricular experiences, how these differences may affect the mathematics they have studied, and their subsequent achievement. Students' opportunities to learn the mathematics covered by the TIMSS Advanced 2008 content and cognitive domains depend initially to some degree on that mathematics being part of each country's guidelines and policies for mathematics education. Thus, participants provided information about various educational policies and the curriculum topics covered in their respective curriculum guidelines (intended curriculum). Inclusion in the country's curriculum, however, does not guarantee students' opportunity to learn. Just as important is what their teachers choose to teach them. The lessons provided by the teachers ultimately determine the mathematics students are taught (implemented curriculum).

The goal of Chapter 1 is to provide information about the teaching and learning of advanced mathematics in each of the 10 countries that participated in the TIMSS Advanced assessment in 2008. It is hoped that this information will enable readers to compare and contrast the different approaches taken by different countries in this area, in order to establish a basis for making cross-country comparisons of outcome data in subsequent chapters.

Among the topics to be covered in Chapter 1 are an overview of the educational systems in the participating countries, descriptions of the populations of students tested, the characteristics of the advanced mathematics curriculum, the amount of time devoted to mathematics in the curriculum at this level, and students' opportunity to learn the advanced mathematics topics covered in the TIMSS Advanced mathematics assessment, including teachers' reports about whether those topics were taught and their feelings about how well prepared they were to teach mathematics at this level.

Overview of the Educational Systems

Mathematics curricula internationally tend to be similar in the early years of schooling.[2] However, at the secondary school level, and especially in the final year or two of secondary school, significant differences can be found across countries in the topics that are included in countries' curricula, in the degree of mathematical rigor expected, in the rates of participation of students in the mathematics courses available at that level, and in the proportions of students still in school and studying advanced mathematics.

Such considerations add to the complexity of making achievement comparisons across countries at this level, but they also heighten the degree of interest in those comparisons. When all children are in school learning the basic concepts and skills of arithmetic, the

2 Mullis, I.V.S., Martin, M.O., & Foy, P. (2008). *TIMSS 2007 international mathematics report: Findings from IEA's Trends in International Mathematics and Science Study at the fourth and eighth grades*. Chestnut Hill, MA: TIMSS & PIRLS International Study Center, Boston College.

basics of geometry, and elementary problem solving, cross-country comparisons, while complicated by socioeconomic and cultural factors among others, are somewhat less problematic. But when there are substantial variations among countries with respect to these kinds of factors, as there are at the senior secondary level, straightforward comparisons are more difficult to draw. Thus, readers of this report are cautioned to be judicious in drawing conclusions about the relative strengths of national systems of education on the basis of the results presented in this volume. The results can be used to examine the range of educational outcomes produced in different countries, and to illustrate the wide range of educational choices that are in effect in those countries.

Exhibit 1.1 presents information about how the overall curriculum for secondary school and the advanced mathematics program are structured in each of the 10 countries that participated in TIMSS Advanced 2008. In 8 of the 10 countries, the last year of secondary school is either the 11th or the 12th year of schooling. The exceptions are Italy where some programs include a 13th year, and the Philippines where the last year of secondary school is the 10th year of schooling. Normally, students in the Russian Federation would complete secondary school after 11 years of schooling; however, about half of the students in their final year at the time of the TIMSS Advanced data collection were in their 10th year, having skipped Year 4 as part of the implementation process for the current program.

In 5 of these 10 countries—Armenia, Iran, Lebanon, the Netherlands, and Sweden—upper secondary schooling consists of a 3-year program. However, in Norway and the Russian Federation it is 2 years, in the Philippines and Slovenia it is 4 years, and in Italy it can be 5 years (the Netherlands may also be considered a 5-year program since it begins with 2 years of basic education where students follow the same curriculum).

The number of hours of advanced mathematics studied was in the range of 100 to 200 hours per year for most countries. It seems clear that students who studied 200 hours or more of mathematics per year (i.e., Iran, Lebanon, and the Russian Federation) would have studied considerably more mathematics in their programs than students in other countries.

In some of the countries, including Armenia, Iran, Italy, the Philippines, and the Russian Federation, students had to meet special entrance requirements (e.g., previous grades, exams, recommendations) to be permitted to enroll in the advanced mathematics program. In the rest of the countries, secondary school students appeared to have considerable latitude in making decisions about which program to follow after completing basic education or general courses required of all students.

In several countries, the students who were identified for participation in TIMSS Advanced 2008 were enrolled in rather highly specialized programs, notably Armenia where the TIMSS advanced mathematics students were enrolled in the "physmat" program and, similarly in Iran, where the track assessed was specifically for university-bound students studying both mathematics and physics. In the Netherlands, most of the TIMSS advanced mathematics students were taking a specialized mathematics program as part of the science and technology program. Those in the Philippines were enrolled in special science and technology schools, and in the Russian Federation they were concentrating on mathematics for 6 hours or more per week in several types of schools. In other countries, a broader cross-section of the final year population was represented.

Exhibit 1.1 **Structural Characteristics of the Advanced Mathematics Programs (Tracks) Assessed by TIMSS Advanced 2008**

TIMSSAdvanced2008 Advanced Mathematics

Country	Description of How the Programs (Tracks) Fit into Overall Curriculum	Number of Years Students Spent in These Programs (Tracks)	Number of Hours of Mathematics Instruction per Year	Criteria for Admission to These Programs (Tracks)
Armenia	Secondary schooling is a 3-year program up to the 11th grade. All students follow the same curriculum through the 3-year program, although students in a small number of special "physmat" schools cover additional topics in mathematics and science. Students at the 11th grade in these "physmat" schools constitute the target population for TIMSS Advanced 2008. As a result of recent reforms to increase the number of years of school, Armenian students were assessed in what is now called the 11th grade. However, since the assessed students skipped a grade as part of implementing the reform, they have had 10 years of formal schooling.	Three years	132	Completion of elementary school and success on the centralized state examination after the 9th grade.
Iran, Islamic Rep. of	After lower secondary schooling (grade 9), students can choose the track they wish to attend in upper secondary school. Students who complete the 11th grade in the mathematics track are allowed to participate in the advanced mathematics and physics track in the pre-university stage. This advanced mathematics and physics track is the target population assessed by TIMSS Advanced 2008.	Three years	220	For enrollment in the advanced mathematics and physics track, students' cumulative grade point average at the 9th grade, their grades in mathematics and science, and the opinion of the school counselor are taken into consideration.
Italy	Secondary education can last 3, 4, or 5 years and is given in four types of schools: lyceums, art schools, technical schools, and vocational schools. The students assessed by TIMSS Advanced 2008 are in grade 13 and have taken an advanced mathematics course or an advanced mathematics and physics course. Most of these students are found in the Liceo Scientifico (general schools with scientific focus), Liceo Scientifico Tecnologico (general school with focus on technology), or Instituti Technici (vocational full time training).	Five years	100	Completion of lower secondary education and success on the national examination after the 8th grade.
Lebanon	Secondary schooling is a 3-year program up to the 12th grade. All students follow the same curriculum in their first year (grade 10). In the second year (grade 11), students can choose between humanities and sciences and in the third year (grade 12), students from the sciences can choose from one of three programs: sociology and economics, life science, or general science. Students from the general science program at the 12th grade constitute the TIMSS Advanced 2008 target population.	Three years	250	Diploma from basic education (brevet).
Netherlands	Secondary education begins with 2 years (grades 7 and 8) of basic education where all students follow the same curriculum. Students can then choose one of three tracks. In the pre-university track (VWO) which is a 4-year program, in the first year (grade 9) all students follow the same curriculum. The next year (grade 10) they can choose one of four programs. Students who select the advanced mathematics course Math B2—most of whom come from the science and technology program—constitute the target population for TIMSS Advanced 2008.	Three years	152*	Students are free to enroll in the different tracks and programs based on their ability and interest.
Norway	The Norwegian students assessed for TIMSS Advanced 2008 had 9 years of compulsory education followed by 3 years of secondary education. The first year of secondary education consists of general courses for all students in the academic track. In the last 2 years, students choose which subjects they want to take. Advanced mathematics courses in the last 2 years consists of 2MX and 3MX. The students assessed by TIMSS Advanced 2008 were in the final year of secondary education and had taken the 3MX mathematics course. After implementing a curriculum reform, the Norwegian school system consists of 13 years of schooling.	Two years	140	Completion of all general courses in the first year of upper secondary schooling.

Data provided by National Research Coordinators.

* Instructional time is not prescribed for advanced mathematics. According to the curriculum, a total of 760 hours over three years should be spent by the students on advanced mathematics (including homework and instruction). About 60% on average should be spent as class time.

Exhibit 1.1 **Structural Characteristics of the Advanced Mathematics Programs (Tracks) Assessed by TIMSS Advanced 2008 (Continued)**

TIMSSAdvanced2008 Advanced Mathematics

Country	Description of How the Programs (Tracks) Fit into Overall Curriculum	Number of Years Students Spent in These Programs (Tracks)	Number of Hours of Mathematics Instruction per Year	Criteria for Admission to These Programs (Tracks)
Philippines	Secondary education is a 4-year program (grades 7–10). Graduates from elementary education may choose to enroll in a general high school or in special schools such as science and technology oriented high schools or in regional science high schools, which prepare students for science-oriented courses in the university. These special schools offer advanced mathematics subjects. Students can also enroll in private and university laboratory high schools, which offer advanced mathematics subjects. Students from these science-oriented schools as well as private and university laboratory high schools offering advanced mathematics subjects are the target population assessed by TIMSS Advanced 2008.	Four years	100-200	Admission to these science high schools may involve a written test, an oral test, and also the grades obtained in elementary school.
Russian Federation	All students study mathematics and physics every year in basic and upper secondary education. In basic education, all students follow the same curriculum, but in upper secondary (grades 10 and 11), the programs differ. The students assessed by TIMSS Advanced 2008 are the 11th grade students who had 6 hours or more per week of instruction in mathematics. These students can be found in lyceums, gymnasiums, special schools for mathematics and physics, and general secondary schools with different profiles in the upper secondary level. As the result of an ongoing reform to increase the number of years of school, Russian students were assessed in what is now called the 11th grade and about half the students have had 11 years of formal schooling. However, the other half skipped grade 4 as part of implementing the reform and only have had 10 years of formal schooling.	Two years	204-306	Admission to the advanced mathematics course involves an interview, students' performance in mathematics for the previous years of schooling, and a written test if necessary.
Slovenia	The Slovenian students assessed for TIMSS Advanced 2008 had 8 years of elementary education and 4 years of secondary education. Secondary education in Slovenia consists of two types of programs: general gymnasia and vocational or technically oriented programs. Only the general gymnasia program offers students the possibility of admission to university studies. All general gymnasia students study the same mathematics courses during their 4-year program. Students in the fourth year of general gymnasia programs were the target population assessed in mathematics by TIMSS Advanced 2008. Currently, Slovenia is in the process of increasing elementary school to 9 years, so that students will have 13 years of schooling.	Four years	105	Completion of elementary schooling. There are no other special admission criteria for the general gymnasia program.
Sweden	Upper secondary education starts from grade 10 and is divided into 17 national 3-year programs. Of these programs, the natural science program has four mandatory mathematics courses (Mathematics A, B, C, and D) and an optional fifth course called Mathematics E. The technology program has three mandatory mathematics courses (Mathematics A, B, and C) and two optional courses (Mathematics D and E). The students assessed by TIMSS Advanced 2008 were the 12th grade students who had taken the Mathematics D course and may have taken the Mathematics E course (58% of students in the sample have taken the Mathematics E course).	Three years	100-150	Completion of compulsory education. Students are then free to choose any upper secondary program.

Data provided by National Research Coordinators.

Description of the Students Assessed for TIMSS Advanced 2008

More information about the makeup of the TIMSS Advanced 2008 target populations in the participating countries can be found in Exhibit 1.2. As noted in the first data column, the number of students in the program or track assessed for TIMSS Advanced 2008 varied from fewer than 3,000 students in Armenia to nearly 120,000 in Italy, primarily because (as described in the introduction) some countries had much larger populations than others. Also, as would be expected based on the variation in the number of years of schooling (shown in the fifth data column), students in their final year of schooling were older in some countries than they were in others, ranging from the relatively young 16-year-old students in the Philippines (with only 10 years of schooling) to those approximately 19 years old in Italy, Norway, Slovenia, and Sweden (with 12 or 13 years of schooling).

Because the number of students taking advanced mathematics in a country is affected not only by the size of the country, but also by the selectivity of the program or track, Exhibit 1.2 provides information about the relative situation in each of the 10 countries. In particular, the TIMSS Advanced Mathematics Coverage Index shown in the fourth data column of Exhibit 1.2 provides a means of comparing the relative sizes of the populations included in the study in these countries. The coverage index for a given country is an estimate of the percentage of the entire national age cohort covered by the TIMSS Advanced target population. It may be helpful to consider the TIMSS Advanced coverage index as a fraction, expressed as a percentage. For most countries, the denominator of the fraction (found in the third data column) is the estimate of the size of the entire national population for the same age cohort as the students tested for TIMSS Advanced. For example, the students assessed in Iran for TIMSS Advanced were,

Exhibit 1.2 **Size of the TIMSS Advanced 2008 Target Population for Advanced Mathematics, the Age Cohort, and the TIMSS Advanced Mathematics Coverage Index**

TIMSSAdvanced 2008 Advanced Mathematics

Country	Estimated Size of the Population of Students in the Final Year of Secondary School Taking the Advanced Mathematics Track or Program Targeted by TIMSS Advanced (Derived from TIMSS Advanced Student Sample)	Age Cohort Corresponding to the Final Year of Secondary School	Size of the Age Cohort Corresponding to the TIMSS Advanced Population Based on National Census Figures[a]	TIMSS Advanced Mathematics Coverage Index – the Percentage of the Entire Corresponding Age Cohort Covered by the TIMSS Advanced Target Population	Years of Formal Schooling*
Armenia	2,684	18	62,758	4.3%	10
Iran, Islamic Rep. of	111,298	18	1,705,000	6.5%	12
Italy	119,162	19	605,507	19.7%	13
Lebanon	4,702	18	79,784	5.9%	12
Netherlands	7,091	18	205,200	3.5%	12
Norway	6,668	19	61,093	10.9%	12
Philippines	14,007	16	1,900,656	0.7%	10
Russian Federation	29,672	17	2,073,041	1.4%	10/11
Slovenia	8,836	19	21,815	40.5%	12
Sweden	16,116	19	125,923	12.8%	12

a Armenia: Estimate derived by dividing the population of 15–19-year olds by 5 for the single year estimate for the year 2008. Data taken from the U.S. Census Bureau's International Database (www.census.gov/). Islamic Rep. of Iran: Total population of 18-year olds in Iran in 2008. Data taken from the Statistical Center of Iran (SCI) (http://www.sci.org.ir/portal/faces/public/sci_en). Italy: Total population of 19-year olds in Italy for the year 2008. Data taken from the Italian Bureau of Statistics (ISTAT) (http://demo.istat.it/pop2008/index.html). Lebanon: Estimate derived by dividing the population of 18–20-year olds by 3 for the single year estimate. Data taken from the Central Bureau for Statistics in the Ministry of Interior. Netherlands: Estimate based on data taken from the Central Bureau of Statistics in the Netherlands (www.cbs.nl). Norway: Total population of 19-year olds in Norway on 1 January 2008. Data taken from the Norwegian National Bureau of Statistics (SSB) (http://www.ssb.no/english/). Philippines: Population of 16-year olds for 2008 projected from the 2000 census. Data taken from the National Statistics Office, Philippines (NSO) (http://www.census.gov.ph/). Russian Federation: Total population of 17-year olds in 2008. Data taken from the Federal State Statistics Service (http://www.gks.ru/wps/portal/english). Slovenia: Estimate was derived by dividing the population of 15–19-year olds by 5 for the single year estimate for the year 2008. Data taken from the Statistical Office of the Republic of Slovenia (www.stat.si). Sweden: Total population of 19-year olds in Sweden for the year 2008. Data taken from Statistics Sweden (SCB) (http://www.scb.se/default____2154.aspx). Data provided by National Research Coordinators.

* Represents years of formal schooling counting from the first year of primary or basic education (first year of ISCED Level1). Because of ongoing reforms in some countries to increase the number of years of schooling, the number of years of formal schooling is not always the same as the grade assessed (see Exhibit 1.1).

on average, 18 years of age (the second data column), so the population estimate for Iran in the third data column is for all 18-year olds in Iran. For Armenia, Lebanon, and Slovenia, data for the age cohorts were not available year-by-year but only for the group of students aged 15 to 19 (18 to 20 for Lebanon), so the population estimates for those countries are averages. The numerator of the fraction is the estimated size of the target population assessed by TIMSS Advanced derived from the TIMSS Advanced student sample (first data column).

The TIMSS Advanced Mathematics Coverage Index expresses the number of students enrolled in the advanced mathematics program or track assessed by TIMSS Advanced as a percentage of all of the students of the same age that could potentially have been in the advanced program or track (if they had all continued their schooling to the final year, wanted to be in the program, and had been accepted). That is, this is the percentage of students in the age cohort in each country receiving the most elite mathematics education. The exhibit shows that the coverage extends from lows of 0.7 and 1.4 percent in the Philippines and the Russian Federation, respectively, to 3.5 and 4.3 percent in the Netherlands and Armenia, to 5.9 and 6.5 percent in Lebanon and Iran, to 10.9 and 12.8 percent in Norway and Sweden, to highs of nearly 20 percent in Italy and 40.5 percent in Slovenia.

The 10 countries that participated in TIMSS Advanced 2008 were very different both in terms of the overall size of their age cohorts (which depend on the size of their national populations), and the numbers of students enrolled in their advanced mathematics programs (which depend both on the size of the population and the degree of selectivity and availability of the program or track assessed). In Iran, the Philippines, and the Russian Federation, the estimated size of the age group from which the TIMSS Advanced 2008 population was selected was greater than 1.5 million. At the opposite extreme, the size of the comparable age cohort in Slovenia was less than 25,000. Armenia, Lebanon, and Norway also had rather small age cohorts, ranging from 60 to 80 thousand.

As has already been indicated, there were large differences across countries in the situations and proportions of the students that were included in the TIMSS Advanced 2008 mathematics study. At one extreme, the three most populous countries—Iran, the Philippines, and the Russian Federation—assessed elite populations of students, as did several countries with much smaller populations, including Armenia, Lebanon, and the Netherlands. In the Philippines, students had fewer years of schooling and were younger than those in the other countries. However, the population assessed for TIMSS Advanced was an elite one for that country, because only a small percentage of students complete secondary schools and those assessed for TIMSS Advanced 2008 were attending the small set of special secondary schools that prepare students for science programs in university, giving them a coverage index of 0.7 percent.

The Russian Federation also assessed an elite population of students. All students in Russia study mathematics and physics every year in lower and upper secondary school. The Russian students assessed for TIMSS Advanced 2008 were from the relatively small percentage who were taking a mathematics course for at least 6 hours a week during the final year of secondary school. This resulted in a coverage index of 1.4 percent. In cases such as these, the rather narrow definition used to define the sample resulted in the selection of a highly specialized group of students compared to other students in the country. And, of course, this fact needs to be borne in mind when making cross-national comparisons.

Some countries elected to assess a much broader cross-section of their students in mathematics. In Slovenia, the smallest participating country in terms of population, there are two types of programs, vocational and general gymnasia, with only the latter offering the possibility of university admission. All students following the general gymnasia program study advanced mathematics and comprised the target population for TIMSS Advanced. This gave Slovenia a coverage index of 41 percent. Italy, with a coverage index of 20 percent, also included a sizeable proportion of their students in their population definition. In Italy, all students who were in Grade 13 and who had taken an advanced mathematics or an advanced mathematics and physics course were included.

Characteristics of the Advanced Mathematics Curriculum

Exhibit 1.3 summarizes how recently the advanced mathematics curriculum has been updated in each of the participating countries. It shows that, in almost all cases, the advanced mathematics curriculum had been revised within the 10 years preceding the TIMSS Advanced 2008 assessment. Several of the participating countries indicated that their advanced mathematics curriculum was in the process of being revised while the data for this study were being collected.

Exhibit 1.4 contains summary information for each country about whether the TIMSS Advanced 2008 mathematics topics were covered in their national curriculum guidelines. The information about topics included in the participants' curricula is discussed in greater depth in Exhibits 1.12 through 1.15, which also include information about the implemented curriculum and provide the results topic-by-topic within each content domain. In general, the countries reported a high degree of correspondence between the topics covered by the TIMSS Advanced 2008 assessment and the topics included in their national curricula for the programs, tracks, or courses identified to be assessed in TIMSS Advanced. As previously described, the framework

Exhibit 1.3 **Structural Characteristics of the Advanced Mathematics Curriculum in Participating Countries**

TIMSSAdvanced2008 Advanced Mathematics

Country	Year Curriculum Taken by Students Assessed in TIMSS Advanced Was Introduced	Curriculum Changes
Armenia	2001	
Iran, Islamic Rep. of	1998	At present there is no national curriculum, instead there are syllabus guides provided by the Mathematics Council of the Organization for Educational Research and Planning, Ministry of Education. Currently, the council is developing the mathematics national curriculum for K–12. In this process the aims, content, teaching, and assessment methods are being revised.
Italy	1923; last revised: Technical Schools 1994, Lyceum 2000	The curriculum is being revised to increase the number of hours of teaching the English language, mathematics and science. The new curriculum will be introduced in 2010.
Lebanon	2001	
Netherlands	1998	The various mathematics subjects have been reorganized and the number of instructional hours reduced from 760 to 600. One new mathematics subject has been added that students can choose; however it is not compulsory. The new curriculum started in August 2007 in grade 10 and therefore has not affected the students participating in TIMSS Advanced 2008.
Norway	2000	A new curriculum was implemented in 2006 with more emphasis on competencies and basic skills and less on instructional methods. The TIMSS Advanced population belonged to the last cohort not affected by this curriculum reform.
Philippines	2004	
Russian Federation	1994 & 2004*	
Slovenia	1998	In 1998, the curriculum for the general gymnasia program was changed to align with the compulsory Matura examination in terms of content, standards, number of hours per subject, and content of compulsory parts of optional courses. The previous curriculum for all 4 years of secondary schools was divided into one curriculum for the general gymnasia program and another curriculum for vocational or technically oriented programs, with the former being more advanced in all subjects.
Sweden	2000	The curriculum is under revision and is intended to be implemented in 2011.

Data provided by National Research Coordinators.

* The Advanced Mathematics classes use as a rule two documents: 1) the syllabus for Advanced Mathematics, introduced in 1994 (not revised since that time); 2) the Educational Standards in Mathematics, introduced in 2004.

Exhibit 1.4 **Number of TIMSS Advanced Mathematics Topics in the Intended Curriculum**

TIMSSAdvanced 2008 Advanced Mathematics

Country	Overall (27 topics)	Algebra (10 Topics)	Calculus (9 topics)	Geometry (8 Topics)
Armenia	22	9	5	8
Iran, Islamic Rep. of	26	9	9	8
Italy	26	9	9	8
Lebanon	27	10	9	8
Netherlands	20	9	7	4
Norway	25	9	9	7
Philippines	25	9	9	7
Russian Federation	25	8	9	8
Slovenia	25	10	8	7
Sweden	19	8	7	4

Data provided by National Research Coordinators.

and the test items for the TIMSS Advanced 2008 mathematics assessment covered three mathematics content domains: algebra, calculus, and geometry. As is shown in Exhibit 1.4, the test items dealt with 27 mathematical topics chosen from the three content domains: 10 in algebra, 9 in calculus, and 8 in geometry.

The vast majority of topics included in the TIMSS Advanced 2008 mathematics framework were included in the advanced mathematics curricula of all the participating countries. In 7 of the 10 countries, almost all (25 or more out of 27) of the topics from the TIMSS Advanced 2008 mathematics framework were included in their intended curriculum. Sweden, the Netherlands, and Armenia had the lowest inclusion rates, the lowest of which was 19 out of 27 in Sweden; that is, an inclusion rate of 70 percent or more across the board. All three content domains had very high inclusion rates, with the rate for geometry being slightly lower than the rate for algebra or calculus. All countries included 8 or more of the 10 algebra topics, and most covered 7 or more of the 9 calculus topics except Armenia (only 5). Most countries also covered either 7 or all 8 of the geometry topics, but the lower rate in this area resulted from the fact that the Netherlands and Sweden had relatively low coverage (half the topics).

Because the TIMSS Advanced assessment attempted to align with instructional practices as much as possible, the assessment was designed so that students could use calculators in ways that mirrored their classroom experiences without unduly advantaging or disadvantaging students either way. Exhibit 1.5 summarizes information concerning the policies in effect in the countries with respect to the use of calculators and computers in mathematics classrooms and during examinations. A majority of participating countries, 8 out of 10, reported permitting students to use calculators of various kinds on national examinations. Two countries, Iran and the Russian Federation, indicated that there

Exhibit 1.5 **Curriculum Studied by TIMSS Advanced Students Includes Policies Regarding Use of Computers and Calculators**

TIMSSAdvanced 2008 Advanced Mathematics

Country	Computers	Calculators	Types of Calculators	Calculators in National Examinations	Description of Policies
Armenia	○	○	○	●	Simple calculators with arithmetic operations are allowed in national examinations.
Iran, Islamic Rep. of	○	○	○	●	Since calculators and computers are not accessible for all students, use of them is not discussed in the national curriculum. Simple calculators only for calculation are permitted in national examination.
Italy	●	○	○	●	There are no policies about the use of calculators, but they are not provided. Students use their own calculator during the examinations. Students use computers while studying some subjects in the lyceum or in specific subjects of technical schools.
Lebanon	○	●	●	●	Non-programmable calculators are permitted. There are no curricular policies about the use of computers. Computer use is optional.
Netherlands	●	●	○	●	Only graphing calculators are allowed in national examinations. The examination board yearly prescribes which brands are allowed.
Norway	●	●	●	●	Graphing calculators are allowed during examinations and frequently used in class. The curriculum, however, only has a vague and general statement about using technological tools in investigations, modeling, and problem solving.
Philippines	●	●	○	○	Information Technology materials/equipment may be used in the teaching/learning process and calculators and computers are considered as IT material/equipment.
Russian Federation	○	○	○	○	There is a general recommendation for middle school that calculators and computers may be used for routine calculations.
Slovenia	●	●	●	●	The national curriculum requires that calculators used in the national examination should be scientific calculators without the capability of symbolic or graphic calculations. During lessons students are allowed to use their own calculators. The use of computers is recommended.
Sweden	○	○	○	●	The students are expected to learn to use graphical, numerical, and symbolic software to find integrals and solve equations; but for the advanced courses it is not stated that this means calculators. National tests in Sweden (which are not formal examinations, but rather tests that are intended to support teachers in their grading of students) are divided into two parts, one where calculators are not allowed, and one where students are expected to have a calculator at hand. The calculators allowed for advanced mathematics students are expected to have a graphing or symbolic capability. There are statements in the curriculum about the use of "Information and Communication Technology", but there are no specific references to computers.

● Yes
○ No

Data provided by National Research Coordinators.

was little, if any, mention of calculator and computer use by students in official documents related to the curriculum. In some countries, curriculum documents encourage teachers to explore applications of technology with their students, but do not provide a lot of specific suggestions or recommendations. Some countries allow students to use graphing calculators during examinations; others forbid their use. In the Netherlands, the examination board each year produces a list of the specific brands of calculators that may be used by students during examinations. On the whole, it seems that mathematics and science educators in many countries are still unsure about how best to incorporate technology into mathematics and science teaching, given the constraints they face in terms of the content of the curriculum and the availability of software of sufficiently high quality and low enough cost to make its adoption possible.

Because public examinations are used in some countries to make decisions about the students enrolled in advanced mathematics programs, tracks, or courses, participating countries were asked to provide information about their examination systems. Exhibit 1.6 indicates that some type of "high-stakes" examinations (i.e., an examination or system of examinations with academic consequences) were a feature of all 10 educational systems except Sweden. In the other participating countries, students write national examinations in mathematics and other subjects during their final year of secondary school and, in some cases, at other grade levels as well. In most cases the important examinations at the end of secondary school are administered by the Ministry of Education or a national examination board. In Sweden, on the other hand, evaluation is the responsibility of the teacher. There are national examinations, but they are intended to supplement the evaluation information that teachers develop on their own.

Exhibit 1.6 Examination System in Participating Countries

TIMSSAdvanced 2008 Advanced Mathematics

Country	Examinations with Consequences for Individuals	Grades at Which Examinations Are Given	Nature and Format of Examination	Purpose of Examination and Consequences	Comments
Armenia	●	Compulsory examinations at grades 9 and 11.		The 9th grade examination is used to determine which students can continue their secondary schooling. The 11th grade examination is necessary for graduation and entry to university.	Both of these are centralized state examinations.
Iran, Islamic Rep. of	●	Examination given at the pre-university year.	Assessment at pre-university includes mid-semester and final examinations.	Passing all subjects in both semesters is a requirement for entering university.	National examinations for grade promotion are given each semester, in two subjects chosen randomly. Examinations in the rest of the subjects are given by the schools. Another national examination is given for entry to the university.
Italy	●	Compulsory examinations at the end of grade 8 and at the end of grade 13.	The assessment includes written tests developed by the teacher and Ministry of Education.	The national examination at grade 8 determines entry to secondary school. The national examination at grade 13 determines entry to university.	Final examinations for technical and professional schools also give students an opportunity to find a job.
Lebanon	●	Examination at the end of the 12th grade.	Written examination.	The examination is used to determine which students have completed secondary schooling and is also used for university admission.	Some university faculties, especially science, engineering and medicine, administer entrance examinations in subjects such as mathematics and physics.
Netherlands	●	There is a national examination at the end of lower-secondary (grade 8) and at the end of upper-secondary education. Depending on the track in upper-secondary the examinations are in grade 10 (pre-vocational), grade 11 (senior general), and grade 12 (pre-university).	Diploma for the upper secondary level is given based on three school-based examinations, number of practical assignments, and final national examinations in different subjects.	The pre-university diploma is needed in order to enter into university.	The national examinations are conducted by the National Examination Board.
Norway	●	Students may be selected for examination in the last 2 years of upper secondary school.	Written national examination or oral local examination.	The examination results influence entrance to tertiary education.	National examinations are administered by the Ministry of Education.
Philippines	●	Schools give achievement tests at the end of every school year for each grade level.	The examinations can be in oral or paper-and-pencil format.	The purpose of the examination is formative. It is used to measure how much a student has learned over a given period of time.	

● Yes
○ No

Data provided by National Research Coordinators.

Exhibit 1.6 **Examination System in Participating Countries (Continued)**

TIMSSAdvanced2008 Advanced Mathematics

Country	Examinations with Consequences for Individuals	Grades at Which Examinations Are Given	Nature and Format of Examination	Purpose of Examination and Consequences	Comments
Russian Federation	●	There is an examination at the end of basic school (grade 9) and at the end of upper secondary school (grade 11).	Examinations in Russian and mathematics are compulsory and conducted in written form.	The purpose of the examination is to certify that students have completed basic education and can enter the next level. The grade 11 examination is necessary for university entrance.	The Federal Service of Supervision in Education and Science administers the examination in mathematics.
Slovenia	●	There is a national examination at the end of elementary school (grade 8) and at the end of secondary school (grade 12). The national examination at the end of secondary school is called the Matura (General Matura for gymnasia program and Vocational Matura for vocational/ technical programs).	The Matura consists of written and oral examinations from the compulsory subjects of mathematics, mother tongue, and foreign language as well as two subjects of the student's choice.	The Matura is a school-leaving examination required for the completion of secondary education and for university entrance.	A pass in the Matura is a general admission requirement for any academic university study program and a minimal admission requirement for those academic courses having no limit on the number of students. Achievement on the Matura and achievement in the last 2 years of schooling are used to select students where there is a limit to the number of candidates for a university program. The Matura is prepared and administered by the National Examination Center.
Sweden	○				Sweden does not have an examination system with direct consequences for individual students. However, national tests are used as an important tool to support teachers in grading their students.

● Yes
○ No

Data provided by National Research Coordinators.

The National Research Coordinators responsible for implementing TIMSS Advanced in each of the participating countries were asked to indicate which of six possible methods for evaluating the degree of implementation of the advanced mathematics curriculum were used in their countries, and their responses are summarized in Exhibit 1.7. The results show that countries tend to use several sources to collect data about curriculum implementation, including results from international comparative studies such as TIMSS Advanced 2008. The most commonly used sources were national examinations, assessments, or tests while the category used least frequently was research and evaluation programs.

All of the participating countries publish either an official curriculum document or a set of notes and directives detailing the advanced mathematics curriculum for teachers, as shown in Exhibit 1.8. Most of them also reported either recommending or mandating particular textbooks to be used by teachers and students for the advanced course. Other kinds of support materials were made available for teachers in some, but not all, countries. These materials included some form of a teacher's guide with suggestions for teaching various topics, suggested instructional activities, and a description of the structure and content of the formal examination to be administered at the end of the year. In some countries, copies of examinations from previous years are made available to teachers and students to familiarize them with the kind of examination they should expect. Armenia, Lebanon, and the Russian Federation indicated that they provide all of these kinds of curriculum support, while Sweden provides only an official curriculum guide for its teachers.

Exhibit 1.9 describes how teachers are kept abreast of changes to the official curriculum in advanced mathematics in their school system. All of the TIMSS Advanced 2008 countries reported documenting such

Exhibit 1.7 **Methods Used to Evaluate the Implementation of the Curriculum for Advanced Mathematics**

TIMSSAdvanced2008 Advanced Mathematics

Country	Visits by Inspectors	Evaluation or Research Programs	School Self-Evaluation	National Examinations, Assessments, or Tests	TIMSS Advanced	Others
Armenia	●	●	●	●	●	● Subject monitored by National Institute of Education
Iran, Islamic Rep. of	●	○	○	●	●	○
Italy	●	○	●	●	●	○
Lebanon	●	●	○	●	●	○
Netherlands	●	○	○	●	○	● Subject monitored and textbooks reviewed by the Netherlands Institute for Curriculum Development (SLO)
Norway	○	●	○	●	●	○
Philippines	●	○	●	●	○	○
Russian Federation	●	●	●	●	●	● Regional monitoring of students' achievement
Slovenia	○	○	●	●	●	○
Sweden	●	○	●	●	●	○

● Yes
○ No

Data provided by National Research Coordinators.

Exhibit 1.8 **Formats in Which the Curriculum for Advanced Mathematics Is Made Available**

TIMSSAdvanced2008 Advanced Mathematics

Country	Official Publication Containing the Curriculum	Ministry Notes and Directives	Mandated or Recommended Textbooks	Instructional or Pedagogical Guide	Specifically Developed or Recommended Instructional Activities	Description of Content of Public Examination	Other
Armenia	●	●	●	●	●	●	○
Iran, Islamic Rep. of	○	●	●	●	○	●	○
Italy	●	●	○	○	●	○	● Professional development for teachers
Lebanon	●	●	●	●	●	●	○
Netherlands	●	●	○	●	●	●	○
Norway	●	●	○	○	○	●	○
Philippines	●	●	○	●	●	○	○
Russian Federation	●	●	●	●	●	●	● Professional development for teachers
Slovenia	●	●	●	○	○	●	● Regular workshops for teachers organized by mathematics department of the National Board for Education
Sweden	●	○	○	○	○	○	○

● Yes
○ No

Data provided by National Research Coordinators.

Exhibit 1.9 **Ways in Which Changes in the Curriculum Are Communicated to Teachers**

TIMSSAdvanced 2008 Advanced Mathematics

Country	Special Conferences/ Seminars	Ministry Website	Printed Copies of the Curriculum Distributed to Schools	Teachers Receive Own Printed Copy	Professional Development/ Inservice Education	Ministry Notes	Professional Association Newsletter	Education Journals	Other*
Armenia	●	●	●	○	●	●	○	●	○
Iran, Islamic Rep. of	●	●	○	○	●	●	●	○	○
Italy	●	●	●	○	●	●	●	●	○
Lebanon	●	●	●	○	●	●	○	○	○
Netherlands	○	●	●	○	●	●	●	●	○
Norway	●	●	●	●	●	●	●	○	○
Philippines	●	●	●	○	●	○	○	○	●
Russian Federation	●	●	○	○	●	●	○	●	○
Slovenia	●	●	○	○	●	○	○	○	●
Sweden	●	●	●	○	●	○	●	●	○

● Yes
○ No

Data provided by National Research Coordinators.

* Phillipines: Information is disseminated through a Department of Education order.
Slovenia: Schools help each other in getting information.

changes on the Ministry of Education's website, conducting special conferences or seminars for teachers (except in the Netherlands), and making various forms of in-service education and professional development opportunities available to teachers. Other activities carried out in five or more countries included distributing copies of revised curricula to schools, issuing notices to schools about recent changes to the curriculum, and publishing announcements of changes in professional association newsletters and in journals for teachers.

Exhibit 1.8 shows that, in Sweden, copies of the official curriculum were made available in printed form to teachers and others, but that none of the other alternatives listed were supported. Exhibit 1.9, on the other hand, shows that Sweden makes use of six of the eight listed alternatives for helping teachers to stay up-to-date with curricular changes. Most countries indicated that they used five or more of the ways listed. The Philippines supported four, and Slovenia, three.

Implementation of the TIMSS Advanced Mathematics Curriculum

Exhibit 1.10 presents information about how many hours of classroom time are devoted each week to advanced mathematics in the participating countries. The National Research Coordinators provided the estimates for the amount of time prescribed in the official curriculum, and the teachers of the students being assessed provided the information about the number of hours devoted to advanced mathematics each week in their own classrooms. While the two estimates were equal only in Norway, there was a fairly high degree of agreement in all countries. That is, the estimate of class time in the intended curriculum is more or less the same as that in the implemented curriculum.

Teachers also were asked to report the percent of instructional time they devoted to the three TIMSS Advanced 2008 content domains—

Exhibit 1.10 **Weekly Hours of Intended and Implemented Instructional Time for Advanced Mathematics in the Final Year**

TIMSSAdvanced 2008 Advanced Mathematics

Country	Intended Instructional Time as Prescribed in the Curriculum (in Hours per Week)	Number of Weeks Schools Are Open in a Year*	Weekly Hours of Implemented Instructional Time for Advanced Mathematics
Armenia	3.9	34	r 4.6 (0.07)
Iran, Islamic Rep. of	6.0	36	4.9 (0.23)
Italy	3.0–5.0	42	3.6 (0.10)
Lebanon	9.6	26	8.6 (0.10)
Netherlands	4.0**	40	3.8 (0.08)
Norway	3.7	38	3.7 (0.07)
Philippines	2.5–5.0	36	5.2 (0.24)
Russian Federation	6.0–9.0	34	5.8 (0.15)
Slovenia	3.7	35	3.8 (0.04)
Sweden	2.7–3.1***	38	3.9 (0.13)

Intended instructional time provided by National Research Coordinators. Implemented instructional time provided by teachers.

* Number of weeks are estimated by dividing total number of school days in a year by five.

** Instructional time is not prescribed for advanced mathematics. According to the curriculum, a total of 760 hours over three years should be spent by the students on advanced mathematics (including homework and instruction). About 60% on average should be spent as class time.

*** Instructional time is not prescribed in the current curriculum. The range above is an estimate based on prescriptions of instructional time from the previous curriculum averaged over three years.

( ) Standard errors appear in parentheses.

An "r" indicates data are available for at least 70% but less than 85% of the students.

algebra, calculus, and geometry—as well as to other topics. As shown in Exhibit 1.11, the three TIMSS Advanced content domains together accounted for at least 80 percent of the instructional time available for advanced mathematics in every country as reported by the final year teachers.

The largest proportion of class time in advanced mathematics in eight of the participating countries was devoted to calculus, but there was considerable variation across countries in this regard. In Italy, 62 percent of class time was taken up by calculus, the largest proportion by far for this group of countries. In Armenia, on the other hand, calculus accounted for only 20 percent of instructional time (presumably because fewer calculus topics are covered as reported in Exhibit 1.4), and the largest segment of the advanced mathematics program there was algebra. Algebra had the highest share of instructional time in the Russian Federation also, although the same percent of time was devoted to geometry as to algebra. Geometry had less time than either of the other two content domains in Italy, the Netherlands, the Philippines, and Slovenia.

TIMSS Advanced asked teachers about the topics actually taught in the mathematics classroom. Teachers of the assessed students were asked to indicate whether each of the TIMSS Advanced topics was *mostly taught before this year*, *mostly taught this year*, or *not yet taught or just introduced*. Exhibit 1.12 presents teachers' reports on the percentages of students who were taught the TIMSS Advanced mathematics topics prior to or during the year of the assessment. The exhibit shows, for each country, averaged across the content domains, the percentage of students whose teachers reported that the students had been taught each topic. Teachers in Lebanon and Slovenia reported an extremely high degree of correspondence, with 95 to 96 percent of the students having been taught the topics. In the remaining countries,

Exhibit 1.11 Percent of Time in Advanced Mathematics Class Devoted to TIMSS Content During the Final Year

TIMSSAdvanced2008 Advanced Mathematics

Country	Algebra	Calculus	Geometry	Other
Armenia	r 37 (0.5)	r 20 (0.8)	r 33 (0.6)	r 10 (0.3)
Iran, Islamic Rep. of	20 (0.9)	41 (1.5)	24 (1.2)	14 (1.0)
Italy	17 (1.1)	62 (1.9)	13 (1.0)	8 (1.4)
Lebanon	r 21 (0.6)	r 35 (0.6)	r 28 (0.5)	r 15 (0.9)
Netherlands	31 (1.2)	34 (1.6)	28 (1.1)	7 (1.2)
Norway	23 (0.9)	31 (1.0)	28 (0.8)	17 (1.0)
Philippines	30 (1.7)	37 (2.1)	27 (1.7)	6 (1.3)
Russian Federation	32 (1.2)	27 (0.8)	32 (0.9)	9 (0.9)
Slovenia	36 (1.1)	43 (1.1)	9 (0.9)	12 (1.2)
Sweden	24 (0.9)	42 (0.8)	32 (1.1)	2 (0.4)

SOURCE: IEA TIMSS Advanced 2008 ©

Data provided by teachers.

( ) Standard errors appear in parentheses.

An "r" indicates data are available for at least 70% but less than 85% of the students.

Exhibit 1.12 Average Percent of Students Taught the TIMSS Advanced 2008 Mathematics Topics Prior to or During the Final Year*

TIMSSAdvanced2008 Advanced Mathematics

Country	Overall (16 topics)	Algebra (6 Topics)	Calculus (5 topics)	Geometry (5 Topics)
Armenia	r 80 (0.4)	r 80 (0.2)	r 66 (1.4)	r 94 (0.2)
Iran, Islamic Rep. of	89 (1.1)	80 (1.2)	95 (1.3)	93 (1.6)
Italy	86 (1.4)	70 (2.0)	94 (1.6)	93 (1.8)
Lebanon	96 (0.4)	95 (0.7)	95 (0.5)	99 (0.2)
Netherlands	86 (0.8)	84 (0.7)	93 (0.9)	81 (1.7)
Norway	89 (0.8)	81 (0.9)	96 (0.9)	91 (1.3)
Philippines	79 (1.6)	85 (1.7)	64 (2.9)	87 (1.5)
Russian Federation	– –	– –	– –	– –
Slovenia	95 (0.6)	97 (0.6)	88 (1.5)	99 (0.4)
Sweden	79 (1.1)	69 (1.5)	95 (1.0)	72 (1.9)

SOURCE: IEA TIMSS Advanced 2008 ©

Data provided by teachers.

* The 27 topics on the intended curriculum in the TIMSS Advanced Curriculum Questionnaire were combined into 16 topics for the Teacher Questionnaire about the implemented curriculum.

A dash (–) indicates comparable data are not available. The Russian Federation did not collect this information.

( ) Standard errors appear in parentheses.

An "r" indicates data are available for at least 70% but less than 85% of the students.

most students (79 to 89 percent) had been taught the topics. Looking at the particular domains, fewer Italian and Swedish students (69–70%) had been taught the algebra topics, fewer Armenian and Philippine students (64–66%) had been taught topics in calculus—Armenia being the country with the least curricular emphasis on this area—and fewer Swedish students (72%) had been taught the geometry topics, consistent with reports about less emphasis on this area in the Swedish curriculum.

As previewed in the discussion of Exhibit 1.4, the participating countries were asked to indicate whether each of the TIMSS Advanced 2008 mathematics topics was included in their intended curriculum; and, as summarized in Exhibit 1.12, the teachers of the TIMSS Advanced 2008 mathematics classes in every country (except the Russian Federation) were asked to indicate whether the advanced mathematics students had been taught that topic. There were 27 topics in all: 10 in algebra, 9 in calculus, and 8 in geometry. The topic-by-topic responses are summarized in Exhibits 1.13 through 1.15.

Exhibit 1.13 shows that 9 of the 10 topics in the algebra domain were reported by the National Research Coordinators to be included in the intended curriculum of their country. The only exception was complex numbers which was included in the intended curriculum of only five countries. As would be anticipated, if the topic of complex numbers was not in the intended curriculum for the country, it was taught to only a few students in those countries. Generally speaking, the remaining TIMSS Advanced topics in algebra corresponded to those topics in the intended curriculum and were taught to a large proportion of the TIMSS Advanced 2008 students.

Exhibit 1.14 shows that all eight of the topics in the TIMSS Advanced 2008 calculus domain were included in the intended curriculum of almost all these countries except Armenia. In particular,

Exhibit 1.13 **Intended and Taught TIMSS Advanced 2008 Algebra Topics**

TIMSSAdvanced2008 Advanced Mathematics

Algebra (10 topics)	Complex Numbers		Numeric and Algebraic Series		Permutations, Combinations, and Probability		
Country	Topic Is in the Intended Curriculum	Percent of Students Taught This Topic	Topic Is in the Intended Curriculum	Percent of Students Taught This Topic	Topic Is in the Intended Curriculum: Permutations and Combinations	Topic Is in the Intended Curriculum: Probability	Average Percent of Students Taught These Topics
Armenia	○	r 12 (0.4)	●	r 95 (0.3)	●	●	r 89 (0.4)
Iran, Islamic Rep. of	○	7 (1.8)	●	93 (2.4)	●	●	93 (2.0)
Italy	●	54 (4.8)	○	34 (4.9)	●	●	46 (5.3)
Lebanon	●	99 (0.3)	●	92 (1.2)	●	●	93 (1.7)
Netherlands	○	14 (3.6)	●	96 (2.3)	●	●	100 (0.0)
Norway	○	3 (1.8)	●	97 (2.0)	●	●	93 (2.4)
Philippines	●	62 (4.9)	●	81 (4.1)	●	●	73 (3.8)
Russian Federation	●	– –	●	– –	●	●	– –
Slovenia	●	100 (0.0)	●	100 (0.0)	●	●	89 (2.7)
Sweden	○	59 (2.4)	●	82 (3.5)	○	●	37 (4.2)

Algebra (10 topics)	Polynomial Equations and Inequalities, Radical Equations, and Logarithmic and Exponential Equations				Equivalent Representation of a Function		Values of a Function and Function of a Function		
Country	Topic Is in the Intended Curriculum: Polynomial Equations and Inequalities	Topic Is in the Intended Curriculum: Radical Equations	Topic Is in the Intended Curriculum: Logarithmic and Exponential Equations	Average Percent of Students Taught These Topics	Topic Is in the Intended Curriculum	Percent of Students Taught This Topic	Topic Is in the Intended Curriculum: Values of a Function	Topic Is in the Intended Curriculum: Function of a Function	Average Percent of Students Taught These Topics
Armenia	●	●	●	r 96 (0.2)	●	r 94 (0.4)	●	●	r 91 (0.3)
Iran, Islamic Rep. of	●	●	●	96 (1.5)	●	95 (1.6)	●	●	96 (1.4)
Italy	●	●	●	97 (1.9)	●	92 (2.7)	●	●	99 (1.3)
Lebanon	●	●	●	97 (1.2)	●	92 (1.4)	●	●	96 (1.4)
Netherlands	●	●	●	100 (0.0)	●	94 (2.0)	●	●	99 (0.8)
Norway	●	●	●	99 (0.6)	●	94 (2.0)	●	●	100 (0.0)
Philippines	●	●	●	97 (1.4)	●	98 (1.2)	●	○	98 (1.3)
Russian Federation	●	●	●	– –	●	– –	●	●	– –
Slovenia	●	●	●	100 (0.0)	●	96 (2.5)	●	●	100 (0.0)
Sweden	●	●	●	98 (1.3)	●	91 (2.7)	●	●	94 (2.3)

● Yes ○ No

Data on intended curriculum provided by National Research Coordinators, and on implemented curriculum by teachers at the time of testing.

A dash (–) indicates comparable data are not available. The Russian Federation did not collect this information.

( ) Standard errors appear in parentheses.

An "r" indicates data are available for at least 70% but less than 85% of the students.

Exhibit 1.14 Intended and Taught TIMSS Advanced 2008 Calculus Topics

TIMSSAdvanced2008 Advanced Mathematics

Calculus (9 topics)	Limits and Continuity			Differentiation of a Function			Using Derivatives to Solve Problems	
Country	Topic Is in the Intended Curriculum: Limits of Functions	Topic Is in the Intended Curriculum: Conditions for Continuity and Differentiability	Average Percent of Students Taught These Topics	Topic Is in the Intended Curriculum: Differentiation of a Function (Polynomial, Logarithmic, Exponential and Trigonometric)	Topic Is in the Intended Curriculum: Differentiation of Composite and Parametric Functions	Average Percent of Students Taught These Topics	Topic Is in the Intended Curriculum	Percent of Students Taught This Topic
Armenia	●	○	r 91 (0.3)	●	●	r 87 (0.4)	●	r 65 (2.3)
Iran, Islamic Rep. of	●	●	97 (1.3)	●	●	97 (1.3)	●	96 (1.3)
Italy	●	●	98 (1.5)	●	●	97 (1.7)	●	94 (2.4)
Lebanon	●	●	99 (0.1)	●	●	97 (1.0)	●	81 (2.0)
Netherlands	○	○	69 (4.6)	●	●	100 (0.0)	●	100 (0.0)
Norway	●	●	84 (4.2)	●	●	98 (1.7)	●	99 (0.7)
Philippines	●	●	88 (2.9)	●	●	81 (3.4)	●	54 (4.8)
Russian Federation	●	●	– –	●	●	– –	●	– –
Slovenia	●	●	100 (0.0)	●	●	100 (0.3)	●	65 (5.7)
Sweden	○	○	77 (4.1)	●	●	100 (0.0)	●	100 (0.5)

Calculus (9 topics)	Gradient, Turning Points, and Points of Inflection of Functions			Integration		
Country	Topic Is in the Intended Curriculum: Using First Derivative to Determine Gradients and Turning Points	Topic Is in the Intended Curriculum: Using Second Derivative to Determine Maxima, Minima, and Points of Inflection	Average Percent of Students Taught These Topics	Topic Is in the Intended Curriculum: Integrating Functions	Topic Is in the Intended Curriculum: Evaluating Definite Integrals	Average Percent of Students Taught These Topics
Armenia	●	○	r 66 (4.3)	○	○	r 20 (4.9)
Iran, Islamic Rep. of	●	●	97 (1.3)	●	●	89 (2.5)
Italy	●	●	96 (2.1)	●	●	85 (3.9)
Lebanon	●	●	100 (0.3)	●	●	98 (0.0)
Netherlands	●	●	100 (0.0)	●	●	98 (1.3)
Norway	●	●	99 (0.6)	●	●	100 (0.0)
Philippines	●	●	57 (4.6)	●	●	41 (5.3)
Russian Federation	●	●	– –	●	●	– –
Slovenia	●	○	93 (1.6)	●	●	81 (3.9)
Sweden	●	●	100 (0.0)	●	●	97 (1.3)

● Yes ○ No

Data on intended curriculum provided by National Research Coordinators, and on implemented curriculum by teachers at the time of testing.

A dash (–) indicates comparable data are not available. The Russian Federation did not collect this information.

( ) Standard errors appear in parentheses.

An "r" indicates data are available for at least 70% but less than 85% of the students.

the Armenian curriculum does not include integration, and few students had been taught these topics. Other than that, four of the topics—differentiation of a function; differentiation of composite and parametric functions; using derivatives to solve problems; and using second derivatives to determine maxima, minima, and points of inflection—were part of the curriculum in every participating country. The others were included in the curricula of at least eight countries. There was a high degree of agreement between the fact that a topic was deemed to be part of the official curriculum for a country and that relatively high percentages of students were taught that topic according to their teachers, but less than in algebra. For example, the Netherlands and Sweden reported that topics related to limits and continuity were not covered in their curricula, yet according to teachers, on average 69 and 77 percent of the students, respectively, had been taught the two topics asked about—limits of functions and conditions for continuity and differentiability. In contrast, the Philippines reported that topics related to derivatives, points of inflection, and integration were included in the curriculum; but according to their teachers only about half of the students or fewer had been taught any of these topics.

Exhibit 1.15 focuses on the geometry content domain where, for many years, there has likely been more variability across countries with respect to which topics should be part of the curriculum and what pedagogical approach should be taken than in any other area of the mathematics curriculum. The topics included in TIMSS Advanced 2008 geometry domain are indicative of that variability, covering quite a wide variety of areas including traditional Euclidean geometry, analytic geometry, transformation geometry approached through vectors, and trigonometry.

Three of the TIMSS Advanced geometry topics—proving geometric propositions in two dimensions, trigonometric properties

Exhibit 1.15 **Intended and Taught TIMSS Advanced 2008 Geometry Topics**

TIMSSAdvanced2008 Advanced Mathematics

Geometry (8 topics)	Properties of Geometric Figures (2- and 3-D)			Gradients, Y–axis Intercepts, and Point of Intersection of Straight Lines in Cartesian Coordinates		Circles		
	Topic Is in the Intended Curriculum					Topic Is in the Intended Curriculum		
Country	Proving Geometric Propositions in 2-D	Proving Geometric Propositions in 3-D	Average Percent of Students Taught These Topics	Topic Is in the Intended Curriculum	Percent of Students Taught This Topic	Equations and Properties of Circles in the Cartesian Plane	Tangents and Normals to Circles	Average Percent of Students Taught These Topics
Armenia	●	●	r 97 (0.1)	●	r 92 (0.3)	●	●	r 95 (0.2)
Iran, Islamic Rep. of	●	●	92 (2.2)	●	95 (1.8)	●	●	93 (2.0)
Italy	●	●	92 (2.9)	●	93 (2.6)	●	●	97 (1.7)
Lebanon	●	●	97 (0.6)	●	99 (0.4)	●	●	100 (0.2)
Netherlands	●	○	100 (0.0)	○	79 (4.9)	○	○	69 (4.7)
Norway	●	●	63 (4.9)	●	99 (0.6)	●	○	92 (3.1)
Philippines	●	●	88 (2.8)	●	96 (1.8)	●	●	98 (1.0)
Russian Federation	●	●	– –	●	– –	●	●	– –
Slovenia	●	○	100 (0.0)	●	100 (0.0)	●	●	96 (2.1)
Sweden	●	○	88 (3.5)	●	97 (1.4)	○	○	55 (5.7)

Geometry (8 topics)	Trigonometry			Properties of Vectors and Their Sums and Differences	
	Topic Is in the Intended Curriculum				
Country	Trigonometric Properties of Triangles	Solving Equations Involving Trigonometric Functions	Average Percent of Students Taught These Topics	Topic Is in the Intended Curriculum	Percent of Students Taught This Topic
Armenia	●	●	r 96 (0.2)	●	r 92 (0.4)
Iran, Islamic Rep. of	●	●	94 (1.8)	●	92 (2.0)
Italy	●	●	98 (1.5)	●	85 (2.7)
Lebanon	●	●	99 (0.3)	●	100 (0.0)
Netherlands	●	●	100 (0.0)	●	55 (4.4)
Norway	●	●	99 (0.6)	●	99 (0.6)
Philippines	●	●	100 (0.2)	○	57 (5.1)
Russian Federation	●	●	– –	●	– –
Slovenia	●	●	100 (0.0)	●	100 (0.0)
Sweden	●	●	100 (0.0)	○	19 (4.5)

● Yes ○ No

Data on intended curriculum provided by National Research Coordinators, and on implemented curriculum by teachers at the time of testing.

A dash (–) indicates comparable data are not available. The Russian Federation did not collect this information.

( ) Standard errors appear in parentheses.

An "r" indicates data are available for at least 70% but less than 85% of the students.

of triangles, and solving equations involving trigonometric functions—were included in the curriculum of all countries and taught to nearly all students. However, in Sweden several geometry topics were not included in the curriculum and only about half the Swedish students were taught geometric topics related to circles and only 19 percent about vectors. Similarly, vectors were not included in the curriculum or taught very much in the Philippines. In a few cases, topics were not considered to be in the curriculum but teachers reported that substantial percentages of the students were taught the topics in any case. For example, in the Netherlands, teachers reported that three geometry topics not specified in the curriculum were taught to large percentages of students: gradients, y-axis intercepts, and intersections in the Cartesian plane (79%); and two circle topics: equations and properties of circles in the Cartesian plane, and tangents and normals to circles (69% on average for the two circle topics).

How Well Prepared Do Teachers Feel They Are to Teach Mathematics?

TIMSS Advanced 2008 asked the students' teachers of mathematics how well prepared they felt they were to teach some of the mathematics topics included in the advanced mathematics framework. For each topic, teachers were asked to indicate whether they felt *very well prepared*, *somewhat prepared*, or *not well prepared*. Teachers were asked about 16 topics in total, including 6 topics in algebra, 5 topics in calculus, and 5 topics in geometry. The percentages of students whose teachers reported feeling very well prepared to teach the various topics are presented in Exhibits 1.16 and 1.17. In Exhibit 1.16, the results are summarized by averaging the percentages of students whose teachers reported feeling very well prepared to teach each topic: first across all of the 16 mathematics topics, and next across the topics in each of

Exhibit 1.16 **Percent of Students Whose Teachers Feel "Very Well" Prepared to Teach the TIMSS Advanced 2008 Mathematics Topics**

TIMSSAdvanced2008
Advanced Mathematics

Country	Percent of Students			
	Overall (16 topics)	Algebra (6 topics)	Calculus (5 topics)	Geometry (5 topics)
Armenia	76 (1.1)	76 (1.7)	66 (2.0)	89 (0.9)
Iran, Islamic Rep. of	87 (1.2)	83 (1.5)	95 (0.9)	84 (2.1)
Italy	69 (2.9)	58 (3.0)	77 (3.6)	71 (3.1)
Lebanon	95 (0.5)	95 (0.6)	92 (0.6)	97 (0.6)
Netherlands	86 (1.8)	84 (2.2)	92 (1.6)	83 (2.6)
Norway	93 (0.9)	87 (1.3)	99 (0.6)	94 (1.2)
Philippines	65 (2.1)	76 (2.3)	51 (3.7)	67 (2.5)
Russian Federation	– –	– –	– –	– –
Slovenia	84 (2.1)	86 (2.5)	79 (3.0)	86 (2.1)
Sweden	81 (1.9)	77 (2.6)	90 (1.7)	77 (2.5)

Data provided by teachers.

A dash (–) indicates comparable data are not available. The Russian Federation did not collect this information.

( ) Standard errors appear in parentheses.

the 3 content domains. Exhibit 1.17 presents the results for each topic. Teachers in the Russian Federation were not asked for this information so the cells in the table for them are blank.

Exhibit 1.16 makes it clear that, in most of the participating countries, the vast majority of students were taught by teachers who considered themselves to be very well prepared to teach these advanced mathematics topics at this level. This result is not particularly surprising, but there may be some cause for concern in those countries where 20 percent or more of the students were taught by teachers who considered themselves either somewhat prepared or not well prepared to teach these 16 topics. Over 90 percent of the advanced mathematics students in Lebanon and Norway as well as more than 80 percent of those in 4 more countries (Iran, the Netherlands, Slovenia, and Sweden) were taught by teachers who considered themselves well prepared, on average, to teach the TIMSS Advanced topics. On the other hand, more than 20 percent of students in Armenia, Italy, and the Philippines were taught by teachers who were not as confident about their degree of preparedness.

Examining these results on a country-by-country and topic-by-topic basis answers some questions about the variability across topics in countries such as Armenia and the Philippines. Exhibit 1.17 shows the percent of students whose teachers considered themselves to be very well prepared to teach the topics in the three TIMSS Advanced 2008 content domains on a topic-by-topic basis, again excluding the Russian Federation. One might expect that almost all mathematics teachers at this level would be well qualified insofar as the subject matter of the course is concerned, and that such teachers would feel themselves to be very well prepared to teach the course. This turns out, surprisingly enough, not to be the case for every topic in every country, and this is reflected in the content of the three tables that make up the exhibit: one table for each of the three content domains.

Exhibit 1.17 **Percent of Students Whose Teachers Feel "Very Well" Prepared to Teach the TIMSS Advanced 2008 Mathematics Topics in Algebra, Calculus, and Geometry**

TIMSSAdvanced2008 Advanced Mathematics

Country	Percent of Students Whose Teachers Report Feeling Very Well Prepared to Teach the Topics in Algebra (6 topics)					
	Complex Numbers	Numeric and Algebraic Series	Permutations, Combinations, and Probability	Polynomial Equations and Inequalities, Radical Equations, and Logarithmic and Exponential Equations	Equivalent Representation of a Function	Values of a Function and Function of a Function
Armenia	40 (4.8)	94 (1.4)	62 (3.7)	94 (1.6)	78 (2.3)	83 (2.1)
Iran, Islamic Rep. of	47 (4.3)	81 (2.9)	80 (3.0)	96 (1.5)	97 (1.2)	96 (1.4)
Italy	54 (4.6)	39 (5.8)	25 (4.3)	87 (3.4)	63 (4.5)	79 (4.3)
Lebanon	98 (0.6)	92 (1.2)	95 (0.9)	96 (1.2)	94 (1.0)	97 (0.8)
Netherlands	61 (5.1)	82 (3.9)	74 (4.5)	98 (1.1)	89 (3.6)	97 (1.4)
Norway	56 (4.7)	99 (0.6)	76 (4.2)	100 (0.0)	94 (2.4)	100 (0.0)
Philippines	82 (3.8)	57 (4.9)	47 (5.1)	91 (3.0)	94 (2.7)	87 (3.3)
Russian Federation	– –	– –	– –	– –	– –	– –
Slovenia	94 (3.5)	89 (4.1)	52 (6.0)	96 (2.1)	89 (3.4)	96 (2.0)
Sweden	80 (3.9)	65 (5.3)	44 (5.5)	96 (1.6)	85 (3.6)	92 (2.9)

Country	Percent of Students Whose Teachers Report Feeling Very Well Prepared to Teach the Topics in Calculus (5 topics)				
	Limits and Continuity	Differentiation of a Function	Using Derivatives to Solve Problems	Gradient, Turning Points, and Points of Inflection of Functions	Integration
Armenia	91 (1.3)	96 (0.2)	52 (4.4)	51 (4.4)	36 (5.5)
Iran, Islamic Rep. of	97 (1.4)	99 (0.6)	88 (2.9)	100 (0.4)	93 (1.7)
Italy	86 (3.7)	84 (3.9)	57 (5.9)	78 (4.7)	79 (4.4)
Lebanon	97 (0.7)	98 (0.7)	70 (2.1)	97 (0.7)	98 (0.6)
Netherlands	82 (3.6)	98 (1.1)	89 (3.4)	98 (1.1)	92 (2.7)
Norway	95 (2.4)	100 (0.0)	97 (1.2)	100 (0.0)	100 (0.2)
Philippines	66 (4.5)	57 (5.4)	34 (5.1)	52 (4.4)	47 (4.7)
Russian Federation	– –	– –	– –	– –	– –
Slovenia	73 (5.1)	95 (2.6)	43 (5.7)	95 (2.7)	89 (4.1)
Sweden	69 (5.3)	97 (1.6)	93 (2.7)	98 (1.3)	95 (2.2)

Data provided by teachers.

A dash (–) indicates comparable data are not available. The Russian Federation did not collect this information.

( ) Standard errors appear in parentheses.

Exhibit 1.17 **Percent of Students Whose Teachers Feel "Very Well" Prepared to Teach the TIMSS Advanced 2008 Mathematics Topics in Algebra, Calculus, and Geometry (Continued)**

TIMSSAdvanced 2008 Advanced Mathematics

Country	Percent of Students Whose Teachers Report Feeling Very Well Prepared to Teach the Topics in Geometry (5 topics)				
	Properties of Geometric Figures (2- and 3-D)	Gradients, Y-axis Intercepts, and Point of Intersection of Straight Lines in Cartesian Coordinates	Circles	Trigonometry	Properties of Vectors and Their Sums and Differences
Armenia	91 (1.7)	85 (1.6)	85 (1.3)	96 (0.2)	87 (3.3)
Iran, Islamic Rep. of	62 (4.2)	91 (2.2)	92 (2.3)	88 (2.2)	86 (2.8)
Italy	49 (5.5)	70 (4.6)	82 (3.9)	87 (3.5)	68 (4.3)
Lebanon	98 (0.6)	98 (0.7)	98 (0.7)	95 (1.0)	98 (0.6)
Netherlands	71 (5.1)	80 (3.3)	79 (4.2)	97 (1.3)	86 (3.8)
Norway	71 (5.2)	100 (0.0)	99 (0.9)	100 (0.4)	99 (1.2)
Philippines	42 (5.3)	87 (3.6)	82 (4.1)	87 (3.8)	39 (5.1)
Russian Federation	– –	– –	– –	– –	– –
Slovenia	59 (5.7)	99 (0.8)	82 (3.7)	94 (2.5)	95 (3.1)
Sweden	63 (5.9)	95 (2.1)	61 (4.8)	97 (1.5)	67 (4.7)

Data provided by teachers.

A dash (–) indicates comparable data are not available. The Russian Federation did not collect this information.

( ) Standard errors appear in parentheses.

If 80 percent or more is used as a criterion for countries where a large majority of students were taught by teachers who consider themselves to be very well prepared to teach a topic, three algebra topics—equations and inequalities, equivalent representations of functions, and functional values and function of a function—are areas that most of these advanced mathematics teachers felt very comfortable teaching. That was not the case for the other three algebra topics: complex numbers, series, and permutations and combinations. In the case of complex numbers, the results are understandable for Armenia, Iran, the Netherlands, Norway, and Sweden, since this topic is not included in their curricula (see Exhibit 1.13) and teachers, therefore, mostly likely do not feel the need to be prepared and may not even be especially trained in this area. On the other hand, for permutations and combinations, the 80 percent or more criterion was reached in fewer than half the countries even though the topic typically was in their curricula. Taking into account that the series topic was not included in the curriculum, the proportion of Italian students who were taught by teachers who considered themselves to be very well prepared to teach these algebra topics was generally lower than in the other countries. Also, as shown in Exhibit 1.13, fewer Swedish students had been taught topics related to probability than might be expected.

The second table in Exhibit 1.17 concerns the five calculus topics, and it raises similar issues. Three topics—differentiation; using derivatives to determine slopes, turning points, and points of inflection of functions; and integrating functions and evaluating definite integrals—are areas in which a large majority of students of advanced mathematics in most countries were being taught by teachers who felt they were well prepared to do so. This, however, was not true for some topics included in the curriculum for some countries (see Exhibit 1.14); for example, limits and continuity in the Netherlands

(69%) and Sweden (77%), and using derivatives to solve problems for several countries. In the latter case, the 80 percent or better criterion was not met in Armenia (65%), the Philippines (54%), and Slovenia (65%), and this should be a particular concern for such an essential part of any introductory calculus course. In particular, the proportions of students who were taught by teachers who considered themselves to be very well prepared to teach the calculus topics were generally lower in the Philippines than in the other countries, and this may help explain why substantial percentages of students were not being taught these topics even though the topics are in the curriculum (see Exhibit 1.14). The percentages also were lower in Armenia, but primarily because a number of the calculus topics (most notably integration) are not in the Armenian curriculum.

The third table in the exhibit deals with the five topics grouped under geometry. The first topic, properties of geometric figures, drew the least support; that is, the percent of students who were taught it by teachers who felt themselves to be well prepared to teach this topic was lower in most countries than for any other topic. The percentages were high (over 90%) only in Armenia and Lebanon, and ranged from 42 to 71 percent for the other countries. It is not clear what one might infer from such a result without knowing more about how teachers interpreted the question they were asked. One possibility is that the inclusion of 3-dimensional figures in the question might have affected the results, but this explanation works best for Sweden where the 3-dimensional topic is not in the curriculum and not covered in all classrooms. For the other countries, this topic was in the curriculum and even when it was not, such as in the Netherlands and Slovenia, teachers unanimously reported teaching it (Exhibit 1.15).

The other four geometry topics had much stronger support. Across the nine countries that provided data, over 80 percent of students in seven or more countries were taught these topics by teachers who considered themselves to be very well prepared to teach these topics. Country-by-country comparisons indicate that the proportions of students who were taught these geometry topics by teachers who felt very well prepared to do so were generally lower in Italy, the Philippines, and Sweden than in the other countries. As observed previously (see Exhibit 1.15), the Swedish curriculum does not include some of the TIMSS Advanced geometry topics. However, all the geometry topics are included in the Italian curriculum and taught to more than 90 percent of the students (except vectors, taught to 80%). Similarly, with the exception of vectors, all of the geometry topics are in the Philippine curriculum and taught to almost all students (88 to 100%). Interestingly, 86 percent of the students in the Netherlands have teachers who feel confident to teach vectors and the topic of vectors is in the Dutch curriculum, but just over half the students (55%) are taught vectors according to their teachers.

In summary, Chapter 1 presents a considerable amount of important information that should be taken into account when considering the achievement results presented in Chapter 2 for each country. Many country characteristics, such as socioeconomic factors and country population size can affect the challenges associated with educating students in advanced mathematics. Beyond that, in some countries, students have had more years of schooling or the advanced mathematics program entails as much as twice the hours of study across the years of the program. In some cases, countries were more selective than others in identifying the students to be assessed in TIMSS Advanced. Also, the curriculum differed somewhat across the advanced mathematics programs assessed in TIMSS Advanced as did teachers' confidence in their preparation to teach the topics assessed. The considerable variation across the 10 participating countries in these system-wide contexts for educating students in advanced mathematics provides a complicated and multifaceted backdrop for considering variation in mathematics achievement.