

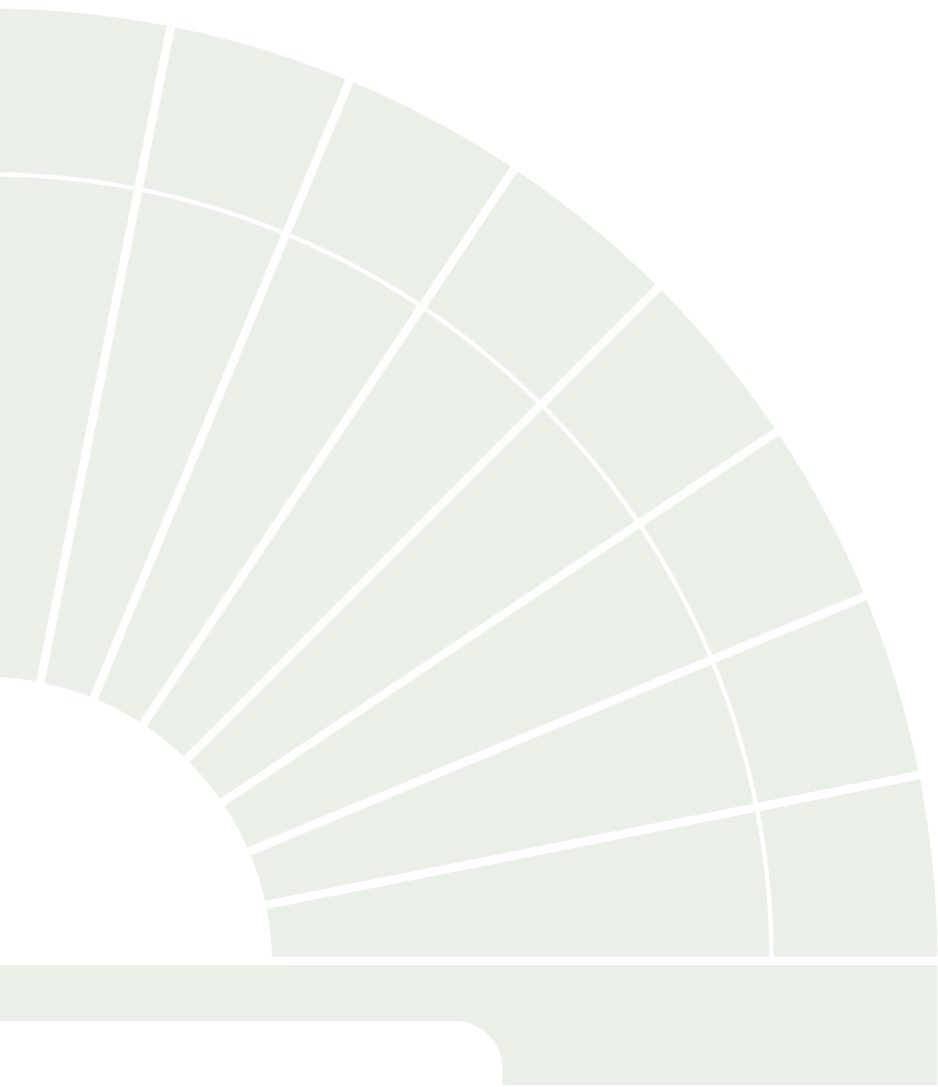
CHAPTER 2

2

Performance at International Benchmarks

The TIMSS 1999 international benchmarks delineate performance of the top 10 percent, top quarter, top half, and lower quarter of students in the countries participating in the study. To help interpret the achievement results, Chapter 2 describes eighth-grade mathematics achievement at each of these benchmarks together with examples of the types of items typically answered correctly by students performing at the benchmark.





As countries around the world spend their time and energy on improving mathematics education, it is important that educators, curriculum developers, and policy makers understand what students know and can do in mathematics and what areas, concepts, and topics need more focus and effort. To help interpret the overall achievement results presented in Chapter 1, this chapter describes eighth-grade mathematics achievement at each of the TIMSS 1999 international benchmarks together with examples of the types of items typically answered correctly by students performing at the benchmark.


Exhibit 1.6, presented previously in Chapter 1, shows the percentages of students in each country reaching each international benchmark – Top 10%, Upper Quarter, Median, and Lower Quarter. The benchmarks delineate performance of the top 10 percent, top quarter, top half, and lower quarter of students in the countries participating in TIMSS 1999 (90th, 75th, 50th, and 25th international percentiles, respectively). The analysis of performance at these benchmarks in mathematics suggests that three primary factors appeared to differentiate performance among the four levels:

- The mathematical operation required
- The complexity of the numbers or number system
- The nature of the problem situation.

For example, there is evidence that students performing at the lower end of the scale could add, subtract, and multiply whole numbers. In contrast, students performing at the higher end of the scale solved non-routine problems involving relationships among fractions, decimals, and percents; various geometric properties; and algebraic rules.

How Were the Benchmark Descriptions Developed?

To develop descriptions of achievement at the TIMSS 1999 international benchmarks, the International Study Center used the scale anchoring method. Scale anchoring is a way of describing students' performance at different points on the TIMSS 1999 achievement scale in terms of the types of items they answer correctly. It involves an empirical component in which items that discriminate between successive points on the scale are identified, and a judgmental component in which subject matter experts examine the content of the items and generalize to students' knowledge and understandings.



For the scale anchoring analysis, the results of students from all the TIMSS 1999 countries were pooled, so that the benchmark descriptions refer to all students achieving at that level. (That is, it does not matter which country the students are from, only how they performed on the test.) Criteria were applied to the TIMSS 1999 achievement scale results to identify the sets of items that students reaching each international benchmark were likely to answer correctly and that those at the next lower benchmark were unlikely to answer correctly.¹ The sets of items produced by the analysis represented the accomplishments of students reaching each successively higher benchmark, and were used by a panel of subject-matter experts from the TIMSS countries to develop the benchmark descriptions.² The work of the panel involved developing a short description for each item of the mathematical understandings demonstrated by students answering it correctly, summarizing students' knowledge and understanding across the set of items for each benchmark to provide more general statements of achievement, and selecting example items illustrating the descriptions.

How Should the Descriptions Be Interpreted?

In general, the parts of the descriptions that relate to the mathematical concepts or familiarity with procedures are relatively straightforward. It needs to be acknowledged, however, that the cognitive behavior necessary to answer some items correctly may vary according to students' experience. An item may require only simple recall for a student familiar with the item's content and context, but necessitate problem-solving strategies from a student unfamiliar with the material. Nevertheless, the descriptions are based on what the panel believed to be the way the great majority of eighth-grade students could be expected to perform when responding to the item.

It also needs to be emphasized that the descriptions of achievement characteristic of students at the international benchmarks are based solely on student performance on the TIMSS 1999 items. Since those items were developed in particular to sample the mathematics domains prescribed for this study, neither the set of items nor the descriptions based on them purport to be comprehensive. There are undoubtedly other mathematics curriculum elements on which students at the various benchmarks would have been successful if they had been included in the assessment.

¹ For example, for the Top 10% Benchmark, an item was included if at least 65 percent of students scoring at the scale point corresponding to this benchmark answered the item correctly and less than 50 percent of students scoring at the Upper Quarter Benchmark answered it correctly. Similarly, for the Upper Quarter Benchmark, an item was included if at least 65 percent of students scoring at that point answered the item correctly and less than 50 percent of students at the Median Benchmark answered it correctly.

² The participants in the scale anchoring process are listed in Appendix E.



Please note that students reaching a particular benchmark demonstrated the knowledge and understandings characterizing that benchmark as well as the competencies of students at the lower benchmarks. The description of achievement at each higher benchmark is cumulative, building on the description of achievement demonstrated by students at the next lower benchmark.

Finally, it must be emphasized that the descriptions of the international benchmarks are provided as one possible way of beginning to examine student performance. Some students scoring below a benchmark may indeed know or understand some of the concepts that characterize a higher level. Thus, it is important to consider performance on the individual items and clusters of items in developing a profile of student achievement in each country.


Several example items are included for each benchmark to complement the descriptions by giving a more concrete notion of the abilities students were able to demonstrate. Each example item is accompanied by the percentage of correct responses for each country as well as the international average. In general, the five or six countries scoring highest on the overall test also scored highest on each of the items used to illustrate benchmarks. Likewise, the five or six countries with the lowest mean achievement also tended to have consistently low percentages of correct responses on the illustrative items. Not surprisingly, this was true for items assessing a range of performance expectations – recall, ability to carry out routine procedures, and ability to solve routine and non-routine problems. The TIMSS 1999 results support the premise that successful problem solving is grounded in mastery of more fundamental knowledge and skills.


Item Examples and Student Performance


The remainder of this chapter describes each benchmark and presents three to five example items illustrating what students know and can do at that level. For each example item, the percent correct for each of the TIMSS 1999 countries is displayed, as well as the international average. The correct answer is circled for multiple-choice items. For open-ended items, the answers shown exemplify the types of student responses that were given full credit. The example items are ones that students reaching each benchmark were likely to answer correctly, and they represent the types of items used to develop the description of achievement at that benchmark.³


³ Some of the items used to develop the benchmark descriptions are being kept secure to measure achievement trends in future TIMSS assessments and are not available for publication.

Achievement at the Top 10% Benchmark

2.1  Exhibit 2.1 describes performance at the Top 10% Benchmark. Students reaching this benchmark demonstrated the ability to organize information in problem-solving situations and to apply their understanding of mathematical relationships. They typically demonstrated success on the knowledge and skills represented by this benchmark, as well as those demonstrated at the Upper Quarter, Median, and Lower Quarter benchmarks.

2.2  Example Item 1 in Exhibit 2.2 illustrates the type of measurement item a student performing at the Top 10% Benchmark generally answered correctly. As can be seen, students had to apply their knowledge of the area of rectangles and inscribed shapes to solve a two-step problem about the area of a garden path. The international average for this item was 42 percent correct. Nevertheless, more than two-thirds of the students answered the item correctly in Hong Kong, Singapore, Japan, Chinese Taipei, and Korea. On average internationally, more than 20 percent of students chose Option A, solving for the area of the larger rectangle rather than that of the path. Option C was an equally popular distracter, with more than 20 percent of students internationally selecting this response.

2.3  Unlike students performing at lower benchmarks, students reaching the Top 10% Benchmark typically could correctly answer multi-step word problems. Example Item 2 in Exhibit 2.3 requires students to select relevant information from two advertisements to solve a complex multi-step word problem involving decimals. Given the price for each issue of a magazine and a certain number of free issues, students were asked to calculate which of the two magazine subscriptions was the less expensive for 24 issues. Students received full credit if they showed correct calculations for at least one of the subscriptions, identified the less expensive magazine, and calculated the difference between the two subscriptions. With an international average of 24 percent correct (for full credit), this item was among the most difficult in TIMSS 1999. Singapore, Korea, and Chinese Taipei were the only countries where the majority of the students answered the item correctly.

2.4  Students reaching the Top 10% Benchmark exhibited an understanding of the properties of similar triangles, as shown by Example Item 3 (see Exhibit 2.4). Given two angle measurements, the length of a side of a triangle, and the dimensions of a second similar triangle, students needed to find the length of an unlabeled side of the first triangle. Internationally, most eighth-grade students had not mastered the concept of proportionality of corresponding sides, or could not solve the resulting

text continued
page 62



• Top 10% Benchmark
Summary

Students can organize information, make generalizations, and explain solution strategies in non-routine problem solving situations. They can organize information and make generalizations to solve problems; apply knowledge of numeric, geometric, and algebraic relationships to solve problems (e.g., among fractions, decimals, and percents; geometric properties; and algebraic rules); and find the equivalent forms of algebraic expressions.

Students can organize information in problem-solving situations. They can select and organize information from two sources to solve a complex word problem involving decimals and organize information to solve a multi-step word problem involving whole numbers.

Students can correctly order the four basic operations in computing with decimals and fractions. Students use their understanding of fractions and decimals in multi-step problem situations. They can solve a problem involving both addition and subtraction of simple common fractions and a problem involving multiplication and subtraction of decimals. They can solve word problems involving fractions and decimals which require analysis of the verbal relations described. They can order a set of decimal fractions of up to three decimal places and can identify the pair of numbers satisfying given conditions involving ordering integers, decimals, and fractions. They can solve a time-distance-rate problem involving decimals and the conversion of minutes to seconds. They can work with part-whole ratios and can solve word problems to find the percent change.

Students can apply their knowledge of measurement in more complex problem situations. They can solve problems involving area and perimeter of rectangles and area of inscribed triangles. They apply knowledge of properties of squares to solve multi-step word problems and draw a new rectangle based on a given rectangle and express the ratio of their areas. They can relate different units of time and apply their knowledge of the number of milliliters in a liter to solve a word problem. They recognize that precision of measurement is related to the size of the unit of measurement.

Students can use their knowledge of angles – overlapping and measures of angles in quadrilaterals – to solve problems. They can use their knowledge of congruent and similar triangles to solve problems concerning corresponding parts. They can identify the coordinates of a point on a line given the coordinates of two other points on the line and locate a point on a number line given its distance from two other points on the line. They can identify the image of a triangle under a rotation in a plane.

Students can use proportion to find missing values in a table. Students can identify an equivalent form of a linear inequality involving a fraction. Students can recognize properties of number operations represented in symbolic form. They can solve a multi-step word problem in which there are two unknowns.

Given the first several terms in pictorial form, that grow in either one or two dimensions, students can make generalizations to find terms in the sequences (e.g. 51st), and they can explain the process used to find those terms.

90th Percentile: 616

equation, with only 37 percent, on average, answering the question correctly. In comparison, top-performing Korea had 70 percent correct responses. Only in Korea, Japan, Singapore, Hong Kong, Chinese Taipei, and Belgium (Flemish) did at least half the students provide the correct solution.

2.5



The eighth-grade students reaching the Top 10% Benchmark typically were able to apply a generalization in order to solve a sequence problem like the one shown in Exhibit 2.5. In this algebra problem, given the initial terms in a sequence and the 50th term of that sequence, they generalized to find the 51st term. This problem was presented in three parts, A, B, and C. For parts A and B, students were asked to indicate how many circles would be in the 5th and 7th figures, respectively, if the pattern were extended. On average internationally, 65 percent of the students answered Part A correctly and 54 percent successfully extended the sequence to the 7th figure in Part B.

To receive full credit for Part C, students had to show or explain how their answer was obtained by providing a general expression or an equation and by calculating the correct number of circles for the 51st figure. Internationally, on average, 30 percent of the students received full credit for their responses. Most of them added the sequence number to the number of circles in the preceding figure: $1275 + 51 = 1326$. Less than three percent of the students internationally calculated the answer by a general expression: $n(n+1)/2$ or $51(52)/2$. About 13 percent of the students in the Netherlands and Moldova received full credit by calculating their answer using the latter method. In 10 countries, 15 percent or less of the students answered Part C of the item correctly. Still, about two-thirds of the students in Korea, Chinese Taipei, Japan, and Singapore received full credit for their responses. It seems worthwhile to note that many students internationally (33 percent) left the item blank, whereas in the four top-performing countries on this item only six to 12 percent of the students did not attempt the item.



Exhibits 2.2–2.5 Overleaf

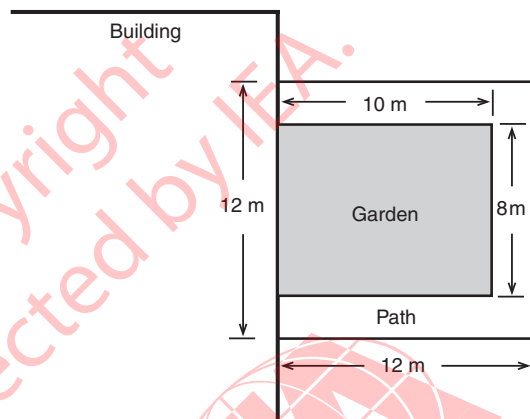
Exhibit 2.2 Top 10% TIMSS International Benchmark – Example Item 1

An Item That Students Reaching the Top 10% International Benchmark Are Likely to Answer Correctly*

Content Area: Measurement

Description: Finds the area between two rectangles when one is inside the other and their sides are parallel.

A rectangular garden that is next to a building has a path around the other three sides, as shown.



What is the area of the path?

- A. 144 m²
- B. 64 m²
- C. 44 m²
- D. 16 m²

	Overall Percent Correct	
Hong Kong, SAR [†]	79 (2.0)	▲
Singapore	78 (2.6)	▲
Japan	74 (1.9)	▲
Chinese Taipei	73 (2.1)	▲
Korea, Rep. of	67 (1.7)	▲
Netherlands [†]	57 (4.4)	▲
Australia	52 (2.6)	▲
Malaysia	52 (2.1)	▲
Slovak Republic	51 (3.3)	●
Canada	51 (3.0)	●
Belgium (Flemish) [†]	51 (2.2)	▲
Finland	46 (3.0)	●
Hungary	46 (2.7)	●
Slovenia	46 (3.2)	●
Cyprus	45 (3.0)	●
Italy	45 (2.7)	●
Bulgaria	42 (3.4)	●
International Avg.	42 (0.4)	
Czech Republic	40 (3.5)	●
England [†]	40 (3.3)	●
New Zealand	40 (2.6)	●
Tunisia	38 (2.0)	●
Russian Federation	38 (3.2)	●
Thailand	35 (2.1)	●
Moldova	34 (2.7)	●
United States	33 (1.6)	▼
Morocco	31 (2.1)	▼
Lithuania ^{1†}	31 (3.0)	▼
Macedonia, Rep. of	30 (2.5)	▼
Romania	29 (2.6)	▼
Jordan	29 (2.3)	▼
Israel ²	28 (2.1)	▼
Latvia (LSS) ¹	28 (2.5)	▼
Iran, Islamic Rep.	26 (2.1)	▼
Indonesia	25 (2.0)	▼
Turkey	22 (1.6)	▼
Chile	18 (1.6)	▼
Philippines	15 (1.2)	▼
South Africa	15 (1.2)	▼

Country average significantly higher than international average ▲

No statistically significant difference between country average and international average ●

Country average significantly lower than international average ▼

Significance tests adjusted for multiple comparisons

SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1998-1999.

* The item was answered correctly by a majority of students reaching this benchmark.

[†] Met guidelines for sample participation rates only after replacement schools were included (see Exhibit A.8).

¹ National Desired Population does not cover all of International Desired Population (see Exhibit A.5). Because coverage falls below 65%, Latvia is annotated LSS for Latvian-Speaking Schools only.

² National Defined Population covers less than 90 percent of National Desired Population (see Exhibit A.5).

[†] Lithuania tested the same cohort of students as other countries, but later in 1999, at the beginning of the next school year.

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

Exhibit 2.3

Top 10% TIMSS International Benchmark – Example Item 2

An Item That Students Reaching the Top 10% International Benchmark Are Likely to Answer Correctly*

Content Area: Data Representation, Analysis and Probability

Description: Selects relevant information from two advertisements to solve a complex word problem involving decimals.

Chris plans to order 24 issues of a magazine. He reads the following advertisements for two magazines. *Ceds* are the units of currency in Chris' country.

Teen Life Magazine

24 issues
First four issues FREE
The rest
3 *ceds* each.

Teen News Magazine

24 issues
First six issues FREE
The rest
3.5 *ceds* each.

Which magazine is the least expensive for 24 issues? How much less expensive? Show your work.

$$\begin{array}{r} \text{Teen Life} = 20 \\ \times 3 \\ \hline 60 \text{ ceds} \\ 24 = 60 \text{ ceds} \end{array}$$

$$\begin{array}{r} \text{Teen News} = 18 \\ \times 3.5 \\ \hline 90 \\ 540 \\ \hline 630 \text{ ceds} \\ 24 = 63 \text{ ceds} \end{array}$$

Teen Life is less expensive by 3 *ceds*.

The answer shown illustrates the type of student response that was given full credit.

	Overall Percent Correct	
Singapore	57 (2.1)	▲
Korea, Rep. of	52 (1.5)	▲
Chinese Taipei	50 (1.8)	▲
Belgium (Flemish) †	42 (1.7)	▲
Japan	39 (1.5)	▲
Slovak Republic	36 (2.3)	▲
Slovenia	36 (2.1)	▲
Hungary	35 (2.1)	▲
Latvia (LSS) †	35 (2.1)	▲
Hong Kong, SAR †	34 (1.8)	▲
Czech Republic	34 (2.5)	▲
Canada	32 (1.8)	▲
Russian Federation	30 (2.4)	●
Australia	29 (2.0)	●
Finland	28 (2.0)	●
Italy	27 (1.7)	●
United States	26 (1.4)	●
Netherlands †	25 (2.7)	●
Lithuania ††	25 (2.0)	●
International Avg.	24 (0.3)	
Bulgaria	22 (2.6)	●
Thailand	21 (1.8)	●
Cyprus	21 (1.8)	●
Romania	20 (2.2)	●
Malaysia	19 (1.4)	▼
Israel †	19 (1.5)	▼
New Zealand	18 (1.7)	▼
Macedonia, Rep. of	17 (1.3)	▼
England †	17 (1.9)	▼
Moldova	16 (1.8)	▼
Jordan	12 (1.1)	▼
Turkey	10 (1.3)	▼
Tunisia	9 (0.8)	▼
Iran, Islamic Rep.	9 (0.7)	▼
Chile	5 (1.0)	▼
Indonesia	5 (0.5)	▼
Philippines	3 (0.7)	▼
Morocco	2 (0.4)	▼
South Africa	1 (0.3)	▼

Country average significantly higher than international average ▲

No statistically significant difference between country average and international average ●

Country average significantly lower than international average ▼

Significance tests adjusted for multiple comparisons

SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1998-1999.

* The item was answered fully correctly by a majority of students reaching this benchmark.

† Met guidelines for sample participation rates only after replacement schools were included (see Exhibit A.8).

‡ National Desired Population does not cover all of International Desired Population (see Exhibit A.5). Because coverage falls below 65%, Latvia is annotated LSS for Latvian-Speaking Schools only.

2 National Defined Population covers less than 90 percent of National Desired Population (see Exhibit A.5).

†† Lithuania tested the same cohort of students as other countries, but later in 1999, at the beginning of the next school year.

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

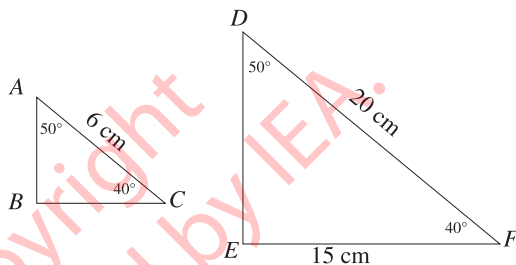
Exhibit 2.4 Top 10% TIMSS International Benchmark – Example Item 3

An Item That Students Reaching the Top 10% International Benchmark Are Likely to Answer Correctly*

Content Area: Geometry

Description: Uses properties of similar triangles to find the length of a corresponding side.

The figure represents two similar triangles. The triangles are not drawn to scale.



In the actual triangle ABC , what is the length of side BC ?

- A. 3.5 cm
- B. 4.5 cm**
- C. 5 cm
- D. 5.5 cm
- E. 8 cm

	Overall Percent Correct	
Korea, Rep. of	70 (1.9)	▲
Japan	68 (1.9)	▲
Singapore	64 (2.7)	▲
Hong Kong, SAR †	56 (2.2)	▲
Chinese Taipei	52 (2.3)	▲
Belgium (Flemish) †	50 (3.2)	▲
Netherlands †	44 (3.1)	●
Hungary	43 (2.9)	●
Russian Federation	41 (2.7)	●
Finland	39 (2.9)	●
Australia	39 (2.8)	●
Romania	38 (2.9)	●
Slovak Republic	38 (3.0)	●
International Avg.	37 (0.4)	
United States	36 (1.6)	●
Moldova	36 (2.4)	●
Canada	35 (2.2)	●
New Zealand	34 (2.7)	●
Slovenia	34 (2.4)	●
England †	34 (2.7)	●
Bulgaria	33 (3.8)	●
Czech Republic	32 (2.5)	●
Malaysia	32 (1.9)	●
Jordan	32 (2.1)	●
Lithuania †*	31 (2.6)	●
Cyprus	31 (2.1)	●
Latvia (LSS) †	30 (2.8)	●
Thailand	30 (1.9)	▼
Italy	29 (2.4)	●
Israel †	29 (2.4)	●
Macedonia, Rep. of	27 (2.5)	▼
Philippines	27 (1.4)	▼
Indonesia	26 (2.0)	▼
Iran, Islamic Rep.	26 (2.1)	▼
Tunisia	24 (1.9)	▼
Chile	23 (1.7)	▼
South Africa	23 (1.3)	▼
Turkey	22 (1.4)	▼

Country average significantly higher than international average ▲

No statistically significant difference between country average and international average ●

Country average significantly lower than international average ▼

Significance tests adjusted for multiple comparisons

SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1998-1999.

* The item was answered correctly by a majority of students reaching this benchmark.

† Met guidelines for sample participation rates only after replacement schools were included (see Exhibit A.8).

¹ National Desired Population does not cover all of International Desired Population (see Exhibit A.5). Because coverage falls below 65%, Latvia is annotated LSS for Latvian-Speaking Schools only.

² National Defined Population covers less than 90 percent of National Desired Population (see Exhibit A.5).

‡ Lithuania tested the same cohort of students as other countries, but later in 1999, at the beginning of the next school year.

Internationally comparable data are unavailable for Morocco.

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

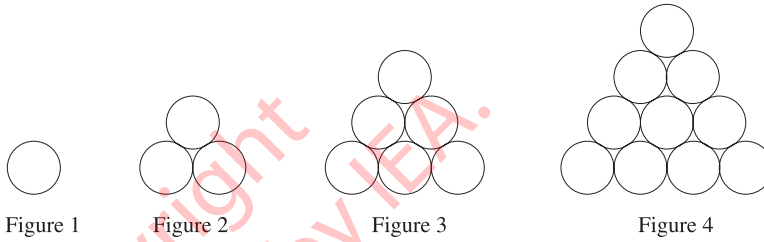
Exhibit 2.5 Top 10% TIMSS International Benchmark – Example Item 4

An Item That Students Reaching the Top 10% International Benchmark Are Likely to Answer Correctly*

Content Area: Algebra

Description: Given the initial terms in a sequence and, for example, the 50th term of that sequence, generalizes to find the next term.

The figures show four sets consisting of circles.



- a) Complete the table below. First, fill in how many circles make up Figure 4. Then, find the number of circles that would be needed for the 5th figure if the sequence of figures is extended.

Figure	Number of circles
1	1
2	3
3	6
4	10
5	15

- b) The sequence of figures is extended to the 7th figure. How many circles would be needed for Figure 7?

Answer: 28

- c) The 50th figure in the sequence contains 1275 circles. Determine the number of circles in the 51st figure. Without drawing the 51st figure, explain or show how you arrived at your answer.

Because it is the 51st figure you have to add 51 to the base of the figure before it

$$\begin{array}{r} 1275 \\ + 51 \\ \hline 1326 \end{array}$$

The answer shown illustrates the type of student response that was given full credit.

	Overall Percent Correct	
Korea, Rep. of	70 (1.2)	▲
Chinese Taipei	68 (1.5)	▲
Japan	66 (1.6)	▲
Singapore	65 (2.4)	▲
Hong Kong, SAR †	57 (2.0)	▲
Netherlands †	48 (3.0)	▲
Belgium (Flemish) †	44 (1.7)	▲
Canada	43 (2.2)	▲
Australia	39 (2.3)	▲
Hungary	38 (1.9)	▲
Malaysia	37 (1.7)	▲
Slovenia	37 (2.3)	●
England †	35 (2.5)	●
United States	34 (1.3)	▲
Czech Republic	34 (2.5)	●
Slovak Republic	31 (2.5)	●
New Zealand	31 (2.0)	●
International Avg.	30 (0.3)	
Finland	30 (2.2)	●
Israel ²	27 (1.6)	●
Russian Federation	27 (2.0)	●
Moldova	26 (2.3)	●
Bulgaria	26 (2.2)	●
Thailand	25 (2.0)	●
Italy	24 (1.8)	▼
Indonesia	24 (1.6)	▼
Latvia (LSS) ¹	22 (2.1)	▼
Romania	19 (2.0)	▼
Lithuania ^{1*}	19 (1.9)	▼
Cyprus	15 (1.5)	▼
Macedonia, Rep. of	13 (1.3)	▼
Jordan	13 (1.3)	▼
Turkey	11 (1.2)	▼
Philippines	9 (0.9)	▼
Chile	8 (1.0)	▼
Tunisia	8 (0.9)	▼
Iran, Islamic Rep.	8 (0.9)	▼
South Africa	3 (0.6)	▼
Morocco	3 (0.5)	▼

Country average significantly higher than international average	▲
No statistically significant difference between country average and international average	●
Country average significantly lower than international average	▼
Significance tests adjusted for multiple comparisons	

SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1998-1999.

* The item was answered fully correctly by a majority of students reaching this benchmark.

† Met guidelines for sample participation rates only after replacement schools were included (see Exhibit A.8).






¹ National Desired Population does not cover all of International Desired Population (see Exhibit A.5). Because coverage falls below 65%, Latvia is annotated LSS for Latvian-Speaking Schools only.

² National Defined Population covers less than 90 percent of National Desired Population (see Exhibit A.5).

† Lithuania tested the same cohort of students as other countries, but later in 1999, at the beginning of the next school year.

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

Achievement at the Upper Quarter Benchmark

- 2.6  Exhibit 2.6 describes performance at the Upper Quarter Benchmark. Eighth-grade students performing at this level applied their mathematical knowledge and understanding in a wide variety of relatively complex problem situations. For example, they demonstrated facility with fractions in a variety of formats, as illustrated by Example Item 5 shown in Exhibit 2.7. This item required students to shade squares in a rectangular grid to represent a given fraction. Since the grid is divided into squares that are a multiple of the fraction's denominator, it requires more than one step to solve the problem. Internationally, about half of the students (49 percent on average) were able to shade in nine of the 24 squares to represent $\frac{3}{8}$ of the region. Eighty percent or more of the students in Singapore, Hong Kong, Belgium (Flemish), Korea, and Chinese Taipei answered the question correctly.
- 2.7  Example Item 6 is a proportional reasoning word problem that students at the Upper Quarter Benchmark typically answered correctly (see Exhibit 2.8). Given the number of magazines sold by each of two boys and the total amount of money made from the sales, students were to calculate how much money one of the boys made by selling his 80 magazines. On average, 44 percent of students internationally answered this question correctly. In Singapore and Chinese Taipei at least three-quarters of the students answered correctly.
- 2.8  Students reaching the Upper Quarter Benchmark generally were able to apply knowledge of geometric properties. In Example Item 7 in Exhibit 2.9, students needed to use their knowledge of the properties of parallelograms and rectangles to solve for the area of the rectangle (dimensions not labeled) that was part of a different figure with given dimensions. Three-quarters or more of the students in Singapore, Japan, Hong Kong, Korea, and Chinese Taipei answered the item correctly. Internationally, however, less than half the eighth-grade students (43 percent on average) did so.
- 2.9  Exhibit 2.10 presents Example Item 8 asking for the number of triangles of a given dimension needed to cover a rectangle of given dimensions. The international average on this item was 46 percent correct. Many students (approximately 29 percent internationally) incorrectly chose Option A, which is half the number of required triangles needed to fill the rectangle but just enough to cover the perimeter. Japanese students
- 2.10 

text continued
page 70



• Upper Quarter Benchmark

Summary

Students can apply their understanding and knowledge in a wide variety of relatively complex situations. They can order, relate and compute with fractions and decimals to solve word problems; solve multi-step word problems involving proportions with whole numbers; solve probability problems; use knowledge of geometric properties to solve problems; identify and evaluate algebraic expressions and solve equations with one variable.

Students demonstrate some facility with fractions and decimals through computation, ordering, rounding, and use in word problems. They can recognize equivalent fractions, add, subtract, multiply and divide fractions with unlike denominators, and correctly order operations. They can identify the smallest decimal from a set of decimals with differing number of places and provide a fraction that is less than a given fraction. They can solve word problems involving multiplication and division of whole numbers and fractions and use pictorial representations of fractions in solving problems. They can identify the fraction of an hour representing a given time interval and identify fractions representing the comparison of part to whole, given each of two parts in a word problem setting.

Students can select the correct rounding of a number involving four decimal places, identify the decimal that is between two decimals given in hundredths, and solve a word problem that involves multiplying a decimal in thousandths by a multiple of a hundred. They can produce an example of a number that would round to a given value. Given a length rounded to the nearest centimeter, they can identify an example of the actual length expressed to one decimal place. Students can identify the ratio expressing a given whole number comparison in a word problem and recognize the effect of adding the same amount to both terms of a ratio. They can estimate products of whole numbers to solve problems. They can solve multi-step word problems involving proportions with whole numbers.

Students demonstrate their understanding of measurement in several settings. They can compare volumes by visualizing and counting cubes. They can calculate the areas of rectangles contained in diagrams of combined shapes. Given the start time and the duration of an event expressed as a fraction of an hour, they can determine the end time. They can estimate the distance between two points on a map, given the scale, and can read unlabeled tick marks on a scale.

Students can use basic properties of triangles, properties of angles on a straight line, and knowledge of symmetry to find the measures of angles. They can identify the angle in a diagram that represents the best estimate of a given measure and recognize that internal angles on a transversal are supplementary. They can visualize the center of a rotation for a two-dimensional figure, the arrangement of faces of a cube when shown its net, and the number of triangles of given dimensions needed to cover a given rectangle. They can identify false statements about congruent triangles and the properties of rectangles.

Students understand elementary concepts of probability, including independent events. They can solve simple problems involving the relationship between successful and unsuccessful outcomes and probabilities. They also recognize that when outcomes are expressed as fractions of a whole, the least likely outcome corresponds to the smallest fraction. They can extrapolate from a graph and determine the number of values on the horizontal axis of a line graph that correspond to a given value on the vertical axis. On a given graph, students can interpolate to find a value between gradations on one axis matching a given value on the other axis.

Students can recognize that multiplication can represent repeated addition. They can identify the algebraic equation corresponding to a verbal description. They can select a simple, multiplicative expression in one variable that is positive for all negative values of the variable. They can substitute numbers for variables to evaluate an expression, and subtract fractions represented algebraically with the same numeric denominator.

Students can solve a linear equation with or without parentheses. They can identify the linear equation that describes the relationship between two variables given in a table of values and select the formula satisfied by the given values of the variables. They can identify the relationship between the first and second terms in a set of ordered pairs.

Given the first several terms of a sequence in pictorial form, growing in either one or two dimensions, they can find specified terms to extend the sequence.

75th Percentile: 555



continued from
page 68

had the highest performance on this item, with 80 percent answering correctly. About two-thirds or more of the students answered the item correctly in Korea, Hong Kong, Singapore, Belgium (Flemish), and the Netherlands.

2.11



Unlike students at lower benchmarks, students reaching the Upper Quarter level typically could solve simple linear equations. As illustrated by Example Item 9 in Exhibit 2.11, for example, students successfully solved for the value of x in a linear equation involving the variable on both sides of the equation. Eighty percent or more of the students in Japan, Hong Kong, and Korea answered this item correctly. On average internationally, 44 percent of students responded correctly.



Exhibits 2.7–2.11 Overleaf

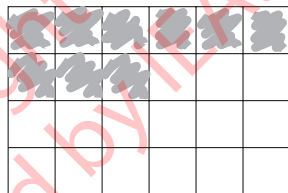
Exhibit 2.7 Upper Quarter TIMSS International Benchmark – Example Item 5

An Item That Students Reaching the Upper Quarter International Benchmark Are Likely to Answer Correctly*

Content Area: Fractions and Number Sense

Description: Shades squares in a rectangular grid to represent a given fraction.

Shade in $\frac{3}{8}$ of the unit squares in the grid.



$$8 \sqrt{24}$$

$$3 \times 3 = 9$$

The answer shown illustrates the type of student response that was given credit.

	Overall Percent Correct	
Singapore	89 (1.7)	▲
Hong Kong, SAR †	87 (1.7)	▲
Belgium (Flemish) †	87 (1.8)	▲
Korea, Rep. of	81 (1.4)	▲
Chinese Taipei	80 (1.9)	▲
Japan	78 (1.9)	▲
Malaysia	73 (2.1)	▲
Canada	68 (2.6)	▲
Finland	65 (2.5)	▲
Hungary	63 (2.5)	▲
Netherlands †	61 (4.7)	●
Australia	60 (2.9)	▲
Slovenia	55 (2.7)	●
Bulgaria	54 (4.3)	●
Cyprus	54 (2.6)	●
England †	52 (2.9)	●
Slovak Republic	52 (3.3)	●
Russian Federation	52 (3.2)	●
United States	49 (1.9)	●
International Avg.	49 (0.4)	
Thailand	49 (2.9)	●
New Zealand	46 (2.9)	●
Italy	46 (2.6)	●
Latvia (LSS) †	46 (2.8)	●
Moldova	44 (3.2)	●
Czech Republic	42 (3.2)	●
Israel †	40 (2.4)	▼
Romania	39 (2.9)	▼
Macedonia, Rep. of	32 (2.4)	▼
Jordan	31 (2.3)	▼
Iran, Islamic Rep.	31 (2.1)	▼
Tunisia	28 (1.8)	▼
Turkey	26 (2.2)	▼
Lithuania ††	26 (2.8)	▼
Indonesia	21 (2.0)	▼
Chile	13 (1.7)	▼
Philippines	11 (1.3)	▼
Morocco	8 (1.1)	▼
South Africa	7 (1.4)	▼
Country average significantly higher than international average		▲
No statistically significant difference between country average and international average		●
Country average significantly lower than international average		▼

Significance tests adjusted for multiple comparisons

SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1998-1999.

* The item was answered correctly by a majority of students reaching this benchmark.

† Met guidelines for sample participation rates only after replacement schools were included (see Exhibit A.8).

† National Desired Population does not cover all of International Desired Population (see Exhibit A.5). Because coverage falls below 65%, Latvia is annotated LSS for Latvian-Speaking Schools only.

2 National Defined Population covers less than 90 percent of National Desired Population (see Exhibit A.5).

† Lithuania tested the same cohort of students as other countries, but later in 1999, at the beginning of the next school year.

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

Exhibit 2.8

Upper Quarter TIMSS International Benchmark – Example Item 6

An Item That Students Reaching the Upper Quarter International Benchmark Are Likely to Answer Correctly*

Content Area: Fractions and Number Sense

Description: Solves a multi-step word problem that involves dividing a quantity in a given ratio.

John sold 60 magazines and Mark sold 80 magazines. The magazines were all sold for the same price. The total amount of money received for the magazines was \$700. How much money did Mark receive?

Answer: \$400

Handwritten student work:

$$\begin{array}{r} 80 \\ \times 60 \\ \hline 140 \text{ TOTAL} \end{array}$$

$$\text{MARK} = \frac{80}{140} = \frac{8}{14} = \frac{4}{7}$$

$$\begin{array}{r} 100 \\ 7 \overline{) 700} \\ \underline{700} \\ 0 \\ \underline{0} \\ 0 \\ \underline{0} \\ 0 \end{array}$$

$$\begin{array}{r} 100 \\ \times 4 \\ \hline \$400 \end{array}$$

The answer shown illustrates the type of student response that was given credit.

	Overall Percent Correct	
Singapore	84 (2.0)	▲
Chinese Taipei	75 (1.8)	▲
Hong Kong, SAR †	72 (2.1)	▲
Korea, Rep. of	69 (1.4)	▲
Japan	67 (2.0)	▲
Malaysia	65 (2.0)	▲
Slovenia	60 (2.7)	▲
Belgium (Flemish) †	60 (3.7)	▲
Hungary	58 (2.5)	▲
Moldova	54 (3.1)	▲
Czech Republic	54 (3.8)	●
Slovak Republic	54 (3.3)	●
Lithuania ††	54 (2.9)	▲
Netherlands †	53 (4.5)	●
Russian Federation	52 (3.1)	●
Bulgaria	50 (3.9)	●
Latvia (LSS) 1	48 (3.4)	●
Finland	47 (3.2)	●
Canada	46 (2.4)	●
International Avg.	44 (0.4)	
Australia	44 (3.2)	●
Romania	43 (3.1)	●
United States	41 (2.0)	●
Cyprus	40 (2.5)	●
Tunisia	39 (2.0)	●
Thailand	38 (2.3)	●
Italy	36 (2.6)	●
New Zealand	33 (2.7)	▼
England †	31 (2.6)	▼
Israel 2	30 (2.5)	▼
Macedonia, Rep. of	30 (2.6)	▼
Iran, Islamic Rep.	28 (2.1)	▼
Indonesia	27 (1.8)	▼
Turkey	26 (1.9)	▼
Jordan	23 (2.0)	▼
Chile	22 (1.7)	▼
Philippines	12 (1.3)	▼
South Africa	9 (1.3)	▼
Morocco	3 (0.6)	▼

Country average significantly higher than international average ▲

No statistically significant difference between country average and international average ●

Country average significantly lower than international average ▼

Significance tests adjusted for multiple comparisons

SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1998-1999.

* The item was answered correctly by a majority of students reaching this benchmark.

† Met guidelines for sample participation rates only after replacement schools were included (see Exhibit A.8).

1 National Desired Population does not cover all of International Desired Population (see Exhibit A.5). Because coverage falls below 65%, Latvia is annotated LSS for Latvian-Speaking Schools only.

2 National Defined Population covers less than 90 percent of National Desired Population (see Exhibit A.5).

† Lithuania tested the same cohort of students as other countries, but later in 1999, at the beginning of the next school year.

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

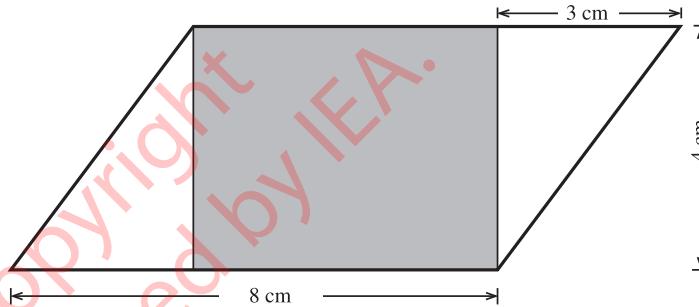
Exhibit 2.9 Upper Quarter TIMSS International Benchmark – Example Item 7

An Item That Students Reaching the Upper Quarter International Benchmark Are Likely to Answer Correctly*

Content Area: Measurement

Description: Finds the area of a rectangle contained in a parallelogram of given dimensions.

The figure shows a shaded rectangle inside a parallelogram.



What is the area of the shaded rectangle?

Answer: 20

$$8 - 3 = 5$$

$$\begin{array}{r} 5 \\ \times 4 \\ \hline 20 \end{array}$$

The answer shown illustrates the type of student response that was given credit.

	Overall Percent Correct	
Singapore	83 (1.5)	▲
Japan	80 (1.2)	▲
Hong Kong, SAR †	78 (1.6)	▲
Korea, Rep. of	78 (1.3)	▲
Chinese Taipei	75 (1.4)	▲
Belgium (Flemish) †	65 (2.0)	▲
Canada	58 (1.6)	▲
Slovak Republic	57 (2.5)	▲
Finland	57 (2.3)	▲
Malaysia	56 (1.9)	▲
Netherlands †	55 (4.7)	●
Australia	55 (1.8)	▲
Bulgaria	52 (3.2)	●
Slovenia	49 (2.1)	●
Russian Federation	49 (2.8)	●
Italy	48 (2.1)	●
England †	48 (2.3)	●
Czech Republic	46 (2.9)	●
Hungary	45 (2.0)	●
Latvia (LSS) †	44 (2.5)	●
International Avg.	43 (0.3)	
Romania	43 (2.7)	●
New Zealand	41 (2.3)	●
Cyprus	41 (1.9)	●
Moldova	38 (2.6)	●
Tunisia	38 (1.6)	▼
Lithuania ††	35 (2.4)	▼
United States	34 (1.4)	▼
Thailand	33 (2.1)	▼
Israel †	28 (1.8)	▼
Jordan	26 (1.5)	▼
Iran, Islamic Rep.	25 (2.0)	▼
Macedonia, Rep. of	25 (1.9)	▼
Turkey	20 (1.7)	▼
Indonesia	20 (1.4)	▼
Morocco	8 (0.9)	▼
Chile	7 (1.2)	▼
Philippines	6 (1.0)	▼
South Africa	3 (0.7)	▼

Country average significantly higher than international average ▲

No statistically significant difference between country average and international average ●

Country average significantly lower than international average ▼

Significance tests adjusted for multiple comparisons

SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1998-1999.

* The item was answered correctly by a majority of students reaching this benchmark.

† Met guidelines for sample participation rates only after replacement schools were included (see Exhibit A.8).

‡ National Desired Population does not cover all of International Desired Population (see Exhibit A.5). Because coverage falls below 65%, Latvia is annotated LSS for Latvian-Speaking Schools only.

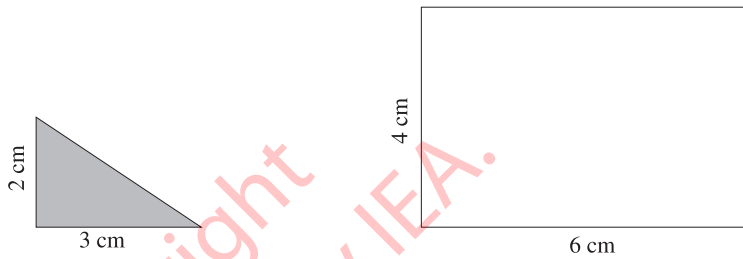
2 National Defined Population covers less than 90 percent of National Desired Population (see Exhibit A.5).

†† Lithuania tested the same cohort of students as other countries, but later in 1999, at the beginning of the next school year.

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

Content Area: Geometry

Description: Determines the number of triangles of given dimensions needed to cover a given rectangle.



How many of the shaded right triangles shown above are needed to exactly cover the surface of the rectangle?

- A. Four
B. Six
C. Eight
D. Ten

	Overall Percent Correct	
Japan	80 (1.8)	▲
Korea, Rep. of	76 (1.7)	▲
Hong Kong, SAR [†]	75 (2.0)	▲
Singapore	72 (2.2)	▲
Belgium (Flemish) [†]	68 (2.7)	▲
Netherlands [†]	66 (3.8)	▲
Malaysia	60 (2.2)	▲
Chinese Taipei	60 (1.8)	▲
Hungary	59 (2.4)	▲
Slovenia	57 (2.6)	▲
Slovak Republic	57 (3.1)	▲
Australia	56 (2.7)	▲
Czech Republic	55 (3.6)	●
New Zealand	55 (2.4)	▲
Canada	50 (2.4)	●
Finland	49 (2.8)	●
Italy	49 (2.7)	●
England [†]	48 (2.6)	●
Latvia (LSS) [‡]	48 (2.9)	●
United States	47 (2.0)	●
International Avg.	46 (0.4)	
Russian Federation	44 (2.8)	●
Lithuania ^{1*}	43 (3.2)	●
Iran, Islamic Rep.	42 (2.1)	●
Israel ²	41 (2.1)	●
Thailand	40 (2.0)	●
Cyprus	37 (3.1)	●
Moldova	37 (2.8)	▼
Romania	35 (2.7)	▼
Bulgaria	34 (3.8)	▼
Tunisia	33 (1.9)	▼
Turkey	30 (1.7)	▼
Macedonia, Rep. of	30 (2.6)	▼
Indonesia	29 (1.9)	▼
Chile	27 (1.8)	▼
Jordan	26 (2.0)	▼
Morocco	21 (1.7)	▼
Philippines	15 (1.4)	▼
South Africa	12 (1.5)	▼

Country average significantly higher than international average ▲

No statistically significant difference between country average and international average ●

Country average significantly lower than international average ▼

Significance tests adjusted for multiple comparisons

SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1998-1999.

* The item was answered correctly by a majority of students reaching this benchmark.

[†] Met guidelines for sample participation rates only after replacement schools were included (see Exhibit A.8).

¹ National Desired Population does not cover all of International Desired Population (see Exhibit A.5). Because coverage falls below 65%, Latvia is annotated LSS for Latvian-Speaking Schools only.

² National Defined Population covers less than 90 percent of National Desired Population (see Exhibit A.5).

[‡] Lithuania tested the same cohort of students as other countries, but later in 1999, at the beginning of the next school year.

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

Exhibit 2.11 Upper Quarter TIMSS International Benchmark – Example Item 9

An Item That Students Reaching the Upper Quarter International Benchmark Are Likely to Answer Correctly*

Content Area: Algebra

Description: Solves a linear equation involving transposing.

Find the value of x if $12x - 10 = 6x + 32$

Answer: 7

$$\begin{aligned}
 12x - 6x - 10 &= 32 \\
 \Rightarrow 6x &= 42 \\
 \Rightarrow \frac{6x}{6} &= \frac{42}{6} \\
 \Rightarrow x &= 7.
 \end{aligned}$$

The answer shown illustrates the type of student response that was given credit.

	Overall Percent Correct	
Japan	85 (1.4)	▲
Hong Kong, SAR [†]	80 (1.9)	▲
Korea, Rep. of	80 (1.5)	▲
Slovak Republic	78 (2.6)	▲
Russian Federation	77 (3.1)	▲
Slovenia	76 (2.8)	▲
Singapore	75 (2.8)	▲
Hungary	74 (2.6)	▲
Chinese Taipei	73 (2.0)	▲
Romania	70 (3.2)	▲
Czech Republic	66 (2.8)	▲
Lithuania ^{1*}	62 (3.4)	▲
Latvia (LSS) ¹	58 (2.9)	▲
Belgium (Flemish) [†]	58 (1.9)	▲
Moldova	56 (3.0)	▲
Macedonia, Rep. of	54 (3.1)	●
Cyprus	51 (3.4)	●
Israel ²	51 (3.1)	●
Italy	46 (2.8)	●
International Avg.	44 (0.4)	
Malaysia	43 (2.7)	●
Bulgaria	34 (3.1)	▼
United States	34 (1.8)	▼
Canada	33 (3.1)	▼
Turkey	32 (2.6)	▼
Australia	31 (3.0)	▼
Thailand	29 (2.8)	▼
England [†]	26 (2.7)	▼
Finland	24 (2.9)	▼
Iran, Islamic Rep.	23 (1.8)	▼
Netherlands [†]	19 (2.9)	▼
New Zealand	19 (2.0)	▼
Jordan	18 (1.9)	▼
Indonesia	18 (2.0)	▼
Chile	12 (1.9)	▼
Morocco	7 (1.0)	▼
Philippines	6 (1.4)	▼
Tunisia	6 (1.0)	▼
South Africa	5 (0.9)	▼

Country average significantly higher than international average ▲

No statistically significant difference between country average and international average ●

Country average significantly lower than international average ▼

Significance tests adjusted for multiple comparisons

SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1998-1999.

* The item was answered correctly by a majority of students reaching this benchmark.

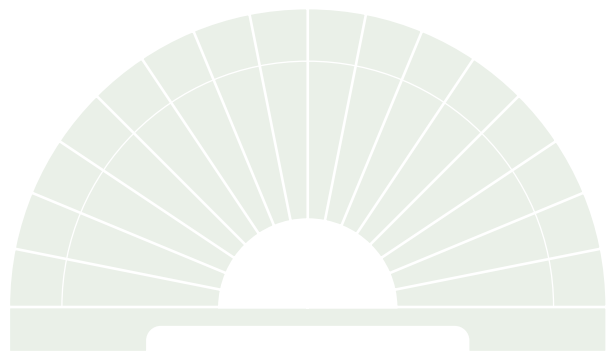
[†] Met guidelines for sample participation rates only after replacement schools were included (see Exhibit A.8).

¹ National Desired Population does not cover all of International Desired Population (see Exhibit A.5). Because coverage falls below 65%, Latvia is annotated LSS for Latvian-Speaking Schools only.

² National Defined Population covers less than 90 percent of National Desired Population (see Exhibit A.5).

[†] Lithuania tested the same cohort of students as other countries, but later in 1999, at the beginning of the next school year.

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.



Achievement at the Median Benchmark

2.12–2.13



Students at the Median Benchmark demonstrated the ability to apply basic mathematical knowledge in straightforward situations (see Exhibit 2.12). For example, as shown by Example Item 10 in Exhibit 2.13, students showed that they understand rounding and can use it to estimate the results of computations. Given the number of rows of cars in a parking lot and the number of cars in each row, students chose the number sentence that would give the best estimate of the total number of cars. While students at the Lower Quarter Benchmark rounded to the nearest hundred, students at the Median Benchmark successfully rounded numbers to get the best estimate for a product. Moreover, middle-performing students demonstrated greater competence with word problems than did those at the Lower Quarter Benchmark. The international average percent correct for this item was 65 percent. Singapore outperformed other countries with 94 percent correct, followed by 85 percent in Hong Kong.

2.14



In geometry, students at the Median Benchmark were able to locate a point on a grid with five-unit divisions where the point lies between the grid lines (see Example Item 11 in Exhibit 2.14). Fifty-eight percent of the students on average internationally correctly chose Point S as the point on the grid that could have the coordinates (7,16). In Japan, Korea, Chinese Taipei, Hong Kong and Singapore, 80 percent or more of the students answered correctly. As might be anticipated, students answering incorrectly most commonly chose Point Q (16,7).

2.15



Example Item 12 shown in Exhibit 2.15 illustrates students' emerging familiarity with algebraic representation. Internationally on average, nearly two-thirds of the students correctly identified the linear equation corresponding to a given verbal statement involving a variable. In Hong Kong, Singapore, Japan, and Korea, 85 percent or more of the students answered correctly.

• Median Benchmark

Summary

Students can apply basic mathematical knowledge in straightforward situations. They can add or subtract to solve one-step word problems involving whole numbers and decimals; identify representations of common fractions and relative sizes of fractions; solve for missing terms in proportions; recognize basic notions of percents and probability; use basic properties of geometric figures; read and interpret graphs, tables, and scales; and understand simple algebraic relationships.

Students can apply basic mathematical knowledge in straightforward situations. They are able to use addition and subtraction to solve one-step word problems involving whole numbers and decimals. They can round whole numbers to the nearest hundred and identify the number sentence that gives the best estimate for the product of two numbers after rounding. Students can arrange four given digits in descending and ascending order to form the largest and smallest possible numbers, and find the difference between those two numbers. Students can approximate the quantity remaining after an amount is reduced by a given percent.

Students demonstrate an understanding of place value in decimal numbers. They can estimate the location of a point representing a decimal number in tenths on a number line marked in whole numbers and identify an unlabeled midway point on a number line marked in tenths. They can set up and solve one-step problems involving addition and subtraction of numbers having up to three decimal places, including situations where the numbers have a different number of decimal places. Given an object of one length, to one decimal place, they can estimate the length of another object.

Students can select the smallest fraction from a list of fractions and can recognize models representing fractions as shaded regions. They can find the missing term in a proportion in word problems and number sentences. Students can solve a simple word problem involving the likelihood of a successful outcome.

Students are able to select the appropriate metric unit to measure the mass of an object. They recognize the inverse relationship between the length of a unit and the number of units required to cover a distance.

Students can locate and interpret data presented in bar graphs, pictographs, pie graphs, and line graphs. Given a table of values for two variables, they can select the graph that represents the given data.

Students can solve problems involving the properties of congruent figures and can select a pair of similar triangles from a set of triangles. They can visualize a rotation of a three-dimensional figure made of cubes. They can locate points in the first quadrant of the Cartesian plane.

Students can select an expression to represent a situation involving multiplication, and identify a linear equation corresponding to a verbal statement. They can find a missing value in a table of values relating x and y values. Using the properties of a balance, they can reason to find an unknown weight. Given diagrams representing the first few terms of a sequence, growing in one dimension, and a partially completed table, they can find the next two terms.

50th Percentile: 479

SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1998-1999.

Exhibit 2.13 Median TIMSS International Benchmark – Example Item 10

An Item That Students Reaching the Median International Benchmark Are Likely to Answer Correctly*

Content Area: Fractions and Number Sense

Description: In a word problem, uses rounding to identify the number sentence that gives the best estimate for the product.

There are 68 rows of cars in a parking lot. Each row has 92 cars. Which of these would give the closest estimate of the total number of cars in the parking lot?

- A. $60 \times 90 = 5400$
 B. $60 \times 100 = 6000$
 C. $70 \times 90 = 6300$
 D. $70 \times 100 = 7000$

	Overall Percent Correct	
Singapore	94 (1.0)	▲
Hong Kong, SAR [†]	85 (1.7)	▲
Belgium (Flemish) [†]	83 (3.0)	▲
Japan	82 (1.4)	▲
Korea, Rep. of	82 (1.2)	▲
Chinese Taipei	81 (1.5)	▲
Netherlands [†]	81 (3.1)	▲
Finland	79 (2.5)	▲
United States	79 (1.8)	▲
Slovak Republic	78 (2.4)	▲
Hungary	78 (2.1)	▲
Canada	78 (2.1)	▲
Czech Republic	78 (2.3)	▲
Malaysia	78 (1.6)	▲
Australia	77 (2.3)	▲
Slovenia	76 (2.5)	▲
England [†]	74 (2.8)	▲
New Zealand	67 (2.6)	●
Russian Federation	65 (2.7)	●
International Avg.	65 (0.4)	
Israel [‡]	63 (2.4)	●
Latvia (LSS) ¹	62 (2.6)	●
Cyprus	60 (2.7)	●
Bulgaria	60 (4.7)	●
Thailand	58 (2.3)	●
Jordan	58 (2.3)	●
Lithuania ^{‡‡}	57 (3.5)	●
Romania	55 (3.0)	●
Macedonia, Rep. of	53 (2.8)	▼
Italy	52 (2.5)	▼
Moldova	52 (2.7)	▼
Turkey	50 (2.0)	▼
Chile	48 (2.4)	▼
Tunisia	48 (2.1)	▼
Iran, Islamic Rep.	48 (2.0)	▼
Indonesia	44 (2.1)	▼
Philippines	42 (1.9)	▼
South Africa	30 (1.8)	▼
Morocco	17 (1.3)	▼
Country average significantly higher than international average		▲
No statistically significant difference between country average and international average		●
Country average significantly lower than international average		▼
Significance tests adjusted for multiple comparisons		

SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1998-1999.

* The item was answered correctly by a majority of students reaching this benchmark.

[†] Met guidelines for sample participation rates only after replacement schools were included (see Exhibit A.8).

¹ National Desired Population does not cover all of International Desired Population (see Exhibit A.5). Because coverage falls below 65%, Latvia is annotated LSS for Latvian-Speaking Schools only.

² National Defined Population covers less than 90 percent of National Desired Population (see Exhibit A.5).

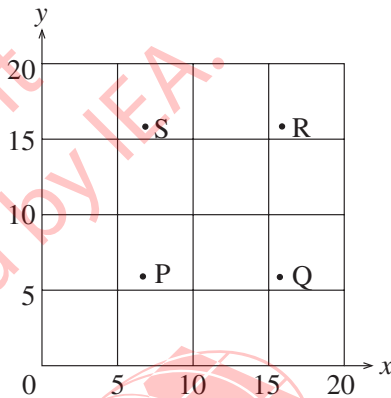
[‡] Lithuania tested the same cohort of students as other countries, but later in 1999, at the beginning of the next school year.

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

Content Area: Geometry

Description: Locates the point on a grid with 5-unit divisions when the point lies between the grid lines.

Which point on the graph could have coordinates (7,16)?



- A. Point P
B. Point Q
C. Point R
D. Point S

	Overall Percent Correct	
Japan	84 (1.7)	▲
Korea, Rep. of	84 (1.4)	▲
Chinese Taipei	83 (1.5)	▲
Hong Kong, SAR [†]	81 (1.7)	▲
Singapore	80 (2.3)	▲
Netherlands [†]	78 (2.5)	▲
Malaysia	78 (1.7)	▲
Slovenia	76 (2.4)	▲
Slovak Republic	76 (2.5)	▲
England [†]	75 (3.2)	▲
Australia	74 (2.3)	▲
Finland	72 (2.7)	▲
New Zealand	72 (2.6)	▲
Hungary	71 (2.5)	▲
Russian Federation	71 (2.2)	▲
Belgium (Flemish) [†]	71 (2.5)	▲
Canada	67 (2.6)	▲
United States	67 (1.6)	▲
Lithuania ^{†‡}	63 (2.9)	●
Italy	62 (2.2)	●
Czech Republic	58 (3.2)	●
International Avg.	58 (0.4)	
Jordan	57 (2.6)	●
Iran, Islamic Rep.	55 (2.2)	●
Bulgaria	53 (2.8)	●
Israel ²	51 (2.7)	●
Indonesia	50 (2.1)	▼
Moldova	48 (2.9)	▼
Romania	47 (2.7)	▼
Latvia (LSS) ¹	46 (2.9)	▼
Macedonia, Rep. of	44 (2.7)	▼
Thailand	37 (2.2)	▼
Turkey	32 (1.9)	▼
Morocco	26 (2.1)	▼
Cyprus	24 (2.1)	▼
Philippines	23 (1.7)	▼
Chile	23 (1.6)	▼
South Africa	20 (1.7)	▼
Tunisia	10 (1.2)	▼

Country average significantly higher than international average ▲

No statistically significant difference between country average and international average ●

Country average significantly lower than international average ▼

Significance tests adjusted for multiple comparisons

SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1998-1999.

* The item was answered correctly by a majority of students reaching this benchmark.

[†] Met guidelines for sample participation rates only after replacement schools were included (see Exhibit A.8).

¹ National Desired Population does not cover all of International Desired Population (see Exhibit A.5). Because coverage falls below 65%, Latvia is annotated LSS for Latvian-Speaking Schools only.

² National Defined Population covers less than 90 percent of National Desired Population (see Exhibit A.5).

[‡] Lithuania tested the same cohort of students as other countries, but later in 1999, at the beginning of the next school year.

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

Exhibit 2.15 Median TIMSS International Benchmark – Example Item 12

An Item That Students Reaching the Median International Benchmark Are Likely to Answer Correctly*

Content Area: Algebra

Description: Identifies the linear equation corresponding to a given verbal statement involving a variable.

n is a number. When n is multiplied by 7, and 6 is then added, the result is 41. Which of these equations represents this relation?

- A. $7n + 6 = 41$
 B. $7n - 6 = 41$
 C. $7n \times 6 = 41$
 D. $7(n + 6) = 41$

	Overall Percent Correct	
Hong Kong, SAR †	93 (0.9)	▲
Singapore	89 (1.7)	▲
Japan	86 (0.8)	▲
Korea, Rep. of	85 (0.7)	▲
Chinese Taipei	84 (1.1)	▲
Slovenia	83 (1.1)	▲
Canada	82 (1.0)	▲
Russian Federation	82 (1.6)	▲
Slovak Republic	81 (1.5)	▲
Belgium (Flemish) †	81 (1.2)	▲
Netherlands †	80 (2.5)	▲
Hungary	80 (1.3)	▲
United States	77 (1.3)	▲
Bulgaria	76 (2.0)	▲
Australia	72 (1.9)	▲
Czech Republic	72 (1.7)	▲
Latvia (LSS) ¹	71 (1.6)	▲
Lithuania ^{1‡}	71 (1.8)	●
Finland	68 (1.5)	●
Israel ²	68 (1.7)	●
Thailand	67 (1.5)	●
Romania	67 (2.1)	●
Cyprus	66 (1.3)	●
International Avg.	65 (0.3)	
Moldova	65 (1.6)	●
Macedonia, Rep. of	63 (1.9)	●
England †	62 (2.1)	●
Italy	58 (1.6)	▼
New Zealand	58 (2.2)	▼
Tunisia	58 (1.4)	▼
Malaysia	57 (1.8)	▼
Jordan	46 (1.4)	▼
Iran, Islamic Rep.	46 (1.5)	▼
Turkey	41 (1.6)	▼
Chile	38 (1.6)	▼
Indonesia	37 (1.4)	▼
Morocco	35 (1.1)	▼
South Africa	21 (1.3)	▼
Philippines	19 (1.6)	▼

Country average significantly higher than international average ▲

No statistically significant difference between country average and international average ●

Country average significantly lower than international average ▼

Significance tests adjusted for multiple comparisons

* The item was answered correctly by a majority of students reaching this benchmark.

† Met guidelines for sample participation rates only after replacement schools were included (see Exhibit A.8).

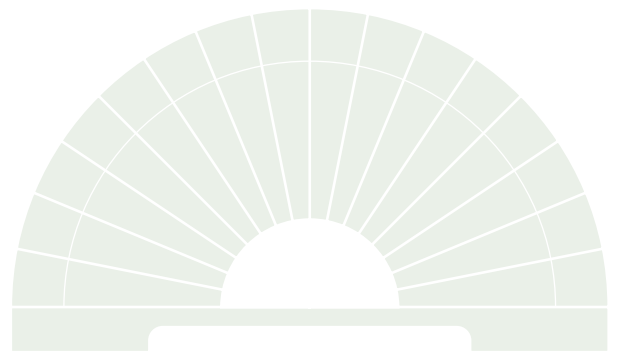
¹ National Desired Population does not cover all of International Desired Population (see Exhibit A.5). Because coverage falls below 65%, Latvia is annotated LSS for Latvian-Speaking Schools only.

² National Defined Population covers less than 90 percent of National Desired Population (see Exhibit A.5).






‡ Lithuania tested the same cohort of students as other countries, but later in 1999, at the beginning of the next school year.

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1998-1999.



Achievement at the Lower Quarter Benchmark

- 2.16  As shown in Exhibit 2.16, the few items anchoring at the Lower Quarter Benchmark provided evidence that students performing at this level can add, subtract, and round with whole numbers. For example, students answering Example Item 13 correctly rounded 691 and 208 to estimate their sum as close to the sum of 700 and 200 (see Exhibit 2.17). The international average was 80 percent correct, and 27 countries had three-quarters or more of their students choosing the correct answer. In four countries – Singapore, Belgium (Flemish), Japan, and the Netherlands – 95 percent or more of the students gave the correct response.
- 2.17  As illustrated by Example Item 14 in Exhibit 2.18, students at the Lower Quarter Benchmark generally could subtract one three-decimal-place number from another with multiple regrouping. Internationally on average, 77 percent of the eighth-grade students selected the correct response to this item. Performance ranged from a high of 92 percent correct in Malaysia to a low of 42 percent correct in South Africa.
- 2.18  Similarly, students at this level could subtract one four-digit integer from another involving multiple regrouping with zeroes (see Example Item 15 in Exhibit 2.19). On this subtraction item also, Malaysia had the highest percentage of students answering this item correctly (94 percent) and South Africa the lowest (37 percent).
- 2.19  In addition, Example Item 16 in Exhibit 2.20 shows that students at this level could read a thermometer and locate the correct reading in a table. There were thirteen countries where at least 90 percent of the students selected the correct response. In only two countries, Turkey and South Africa, did less than 50 percent of the students answer the item correctly.
- 2.20 

• Lower Quarter Benchmark**Summary**

Students can do basic computations with whole numbers.

The few items at this level provide some evidence that students can add, subtract, and round with whole numbers. When there are the same number of decimal places, they can subtract with multiple regrouping. Students can round whole numbers to the nearest hundred. They can read a thermometer and locate the reading in a table. Students recognize some basic notation.

25th Percentile: 396

SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1998-1999.

Exhibit 2.17 Lower Quarter TIMSS International Benchmark – Example Item 13

An Item That Students Reaching the Lower Quarter International Benchmark Are Likely to Answer Correctly*

Content Area: Fractions and Number Sense

Description: Rounds to estimate the sum of two three-digit numbers.

The sum $691 + 208$ is closest to the sum

- A. $600 + 200$
 B. $700 + 200$
 C. $700 + 300$
 D. $900 + 200$

	Overall Percent Correct	
Singapore	97 (0.5)	▲
Belgium (Flemish) †	96 (0.7)	▲
Japan	95 (0.5)	▲
Netherlands †	95 (0.8)	▲
Hong Kong, SAR †	93 (0.7)	▲
Canada	93 (0.7)	▲
United States	93 (0.7)	▲
Hungary	93 (0.9)	▲
Korea, Rep. of	93 (0.6)	▲
Slovenia	92 (0.8)	▲
England †	92 (1.0)	▲
Czech Republic	91 (1.0)	▲
Australia	91 (0.8)	▲
Finland	91 (1.0)	▲
Slovak Republic	90 (1.1)	▲
Chinese Taipei	89 (0.7)	▲
New Zealand	88 (1.0)	▲
Malaysia	88 (0.8)	▲
Latvia (LSS) 1	87 (1.4)	▲
Bulgaria	86 (1.6)	▲
Cyprus	85 (1.1)	▲
Lithuania 1†	84 (1.5)	●
Russian Federation	83 (1.9)	●
Israel 2	83 (1.6)	●
International Avg.	80 (0.2)	
Macedonia, Rep. of	79 (1.4)	●
Italy	77 (1.9)	●
Thailand	77 (1.5)	●
Turkey	74 (1.3)	▼
Romania	73 (1.8)	▼
Tunisia	67 (1.3)	▼
Jordan	66 (1.5)	▼
Moldova	66 (1.6)	▼
Chile	65 (1.3)	▼
Iran, Islamic Rep.	58 (1.5)	▼
Indonesia	54 (1.6)	▼
Philippines	53 (1.6)	▼
Morocco	43 (1.2)	▼
South Africa	37 (1.6)	▼

Country average significantly higher than international average ▲

No statistically significant difference between country average and international average ●

Country average significantly lower than international average ▼

Significance tests adjusted for multiple comparisons

* The item was answered correctly by a majority of students reaching this benchmark.

† Met guidelines for sample participation rates only after replacement schools were included (see Exhibit A.8).

1 National Desired Population does not cover all of International Desired Population (see Exhibit A.5). Because coverage falls below 65%, Latvia is annotated LSS for Latvian-Speaking Schools only.

2 National Defined Population covers less than 90 percent of National Desired Population (see Exhibit A.5).

† Lithuania tested the same cohort of students as other countries, but later in 1999, at the beginning of the next school year.

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

Content Area: Fractions and Number Sense

Description: Subtracts a three-decimal-place number from another with multiple regrouping.

Subtract: $4.722 - 1.935 =$

- (A) 2.787
 B. 2.797
 C. 2.887
 D. 2.897

	Overall Percent Correct	
Malaysia	92 (1.1)	▲
Singapore	90 (1.4)	▲
Hungary	90 (1.7)	▲
Slovenia	90 (1.6)	▲
Korea, Rep. of	88 (1.2)	▲
Russian Federation	88 (1.9)	▲
Slovak Republic	87 (2.1)	▲
Japan	86 (1.3)	▲
Lithuania ^{††}	86 (2.1)	▲
Czech Republic	85 (2.8)	●
Chinese Taipei	84 (1.5)	▲
Hong Kong, SAR [†]	83 (1.8)	▲
Thailand	83 (1.6)	▲
Tunisia	82 (1.6)	●
Bulgaria	81 (2.6)	●
Moldova	80 (2.3)	●
Canada	80 (1.8)	●
Latvia (LSS) [†]	79 (2.4)	●
Indonesia	78 (1.9)	●
Romania	77 (2.5)	●
United States	77 (1.7)	●
Italy	77 (2.3)	●
International Avg.	77 (0.4)	
Chile	75 (1.7)	●
Australia	74 (2.7)	●
Belgium (Flemish) [†]	73 (2.0)	●
Finland	72 (3.0)	●
Cyprus	71 (2.2)	●
Macedonia, Rep. of	71 (2.4)	●
Iran, Islamic Rep.	71 (2.3)	●
Turkey	71 (1.9)	●
Netherlands [†]	69 (4.3)	●
Philippines	69 (1.8)	▼
Jordan	65 (2.4)	▼
Israel ²	63 (2.5)	▼
Morocco	62 (2.5)	▼
New Zealand	61 (2.5)	▼
England [†]	59 (2.7)	▼
South Africa	42 (1.8)	▼

Country average significantly higher than international average ▲

No statistically significant difference between country average and international average ●

Country average significantly lower than international average ▼

Significance tests adjusted for multiple comparisons

SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1998-1999.

* The item was answered correctly by a majority of students reaching this benchmark.

† Met guidelines for sample participation rates only after replacement schools were included (see Exhibit A.8).

‡ National Desired Population does not cover all of International Desired Population (see Exhibit A.5). Because coverage falls below 65%, Latvia is annotated LSS for Latvian-Speaking Schools only.

2 National Defined Population covers less than 90 percent of National Desired Population (see Exhibit A.5).

†† Lithuania tested the same cohort of students as other countries, but later in 1999, at the beginning of the next school year.

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

Content Area: Fractions and Number Sense

Description: Subtracts a four-digit number from another involving zeroes.

Subtract:

$$\begin{array}{r} 7003 \\ - 4078 \\ \hline \end{array}$$

A. 2035

B. 2925

C. 3005

D. 3925

	Overall Percent Correct	
Malaysia	94 (0.9)	▲
Singapore	92 (1.3)	▲
Chinese Taipei	90 (1.2)	▲
Hong Kong, SAR †	90 (1.3)	▲
Korea, Rep. of	88 (1.2)	▲
Hungary	87 (1.8)	▲
Slovak Republic	86 (1.9)	▲
Japan	86 (1.4)	▲
Belgium (Flemish) †	85 (2.1)	▲
Slovenia	83 (2.2)	▲
Canada	83 (1.4)	▲
Czech Republic	82 (2.4)	●
United States	81 (1.6)	▲
Lithuania †*	80 (2.7)	●
Tunisia	80 (1.7)	▲
Russian Federation	79 (2.2)	●
Moldova	79 (2.2)	●
Netherlands †	79 (3.4)	●
Australia	77 (2.5)	●
Thailand	77 (1.8)	●
Finland	76 (2.4)	●
Bulgaria	76 (2.9)	●
International Avg.	74 (0.4)	
Latvia (LSS) †	74 (3.1)	●
Iran, Islamic Rep.	73 (1.9)	●
Cyprus	70 (2.2)	●
Turkey	69 (1.9)	●
Jordan	69 (2.1)	●
Romania	68 (2.9)	●
Israel †	67 (2.4)	●
Italy	67 (2.7)	●
Macedonia, Rep. of	65 (2.7)	▼
Chile	59 (2.0)	▼
Philippines	58 (1.9)	▼
New Zealand	58 (2.4)	▼
Indonesia	55 (2.6)	▼
Morocco	54 (2.1)	▼
England †	51 (3.1)	▼
South Africa	37 (2.0)	▼

Country average significantly higher than international average ▲

No statistically significant difference between country average and international average ●

Country average significantly lower than international average ▼

Significance tests adjusted for multiple comparisons

SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1998-1999.

* The item was answered correctly by a majority of students reaching this benchmark.

† Met guidelines for sample participation rates only after replacement schools were included (see Exhibit A.8).

‡ National Desired Population does not cover all of International Desired Population (see Exhibit A.5). Because coverage falls below 65%, Latvia is annotated LSS for Latvian-Speaking Schools only.

§ National Defined Population covers less than 90 percent of National Desired Population (see Exhibit A.5).

¶ Lithuania tested the same cohort of students as other countries, but later in 1999, at the beginning of the next school year.

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

Content Area: Data Representation, Analysis and Probability

Description: Reads a thermometer and locates the reading in a table.

This table shows temperatures at various times on four days.

TEMPERATURE					
	6 a.m.	9 a.m.	Noon	3 p.m.	6 p.m.
Monday	15°	17°	24°	21°	16°
Tuesday	20°	16°	15°	10°	9°
Wednesday	8°	14°	16°	19°	15°
Thursday	8°	11°	19°	26°	20°



Thermometer

On which day and at what time was the temperature shown in the table the same as that shown on the thermometer.

- A. Monday, Noon
 B. Tuesday, 6 a.m.
 C. Wednesday, 3 p.m.
 D. Thursday, 3 p.m.

	Overall Percent Correct	
Japan	96 (0.8)	▲
Singapore	95 (0.9)	▲
Belgium (Flemish) †	95 (1.5)	▲
Finland	93 (1.4)	▲
Korea, Rep. of	92 (0.9)	▲
England †	92 (2.2)	▲
Chinese Taipei	91 (1.2)	▲
Slovenia	91 (1.7)	▲
Czech Republic	91 (1.9)	▲
Australia	91 (2.2)	▲
Slovak Republic	91 (1.5)	▲
Hong Kong, SAR †	90 (1.5)	▲
Netherlands †	90 (2.6)	▲
Canada	89 (2.6)	▲
United States	89 (1.2)	▲
New Zealand	88 (1.9)	▲
Hungary	87 (2.0)	▲
Cyprus	86 (1.4)	▲
Russian Federation	85 (2.6)	●
Malaysia	85 (1.4)	▲
Lithuania ††	84 (2.4)	●
Latvia (LSS) 1	83 (2.3)	●
Italy	81 (2.0)	●
International Avg.	79 (0.3)	
Israel 2	74 (2.0)	●
Bulgaria	72 (2.8)	●
Chile	67 (1.9)	▼
Moldova	66 (2.8)	▼
Romania	65 (2.8)	▼
Jordan	65 (1.9)	▼
Macedonia, Rep. of	65 (2.9)	▼
Iran, Islamic Rep.	59 (2.5)	▼
Philippines	54 (2.0)	▼
Indonesia	50 (2.3)	▼
South Africa	43 (2.1)	▼
Turkey	38 (1.9)	▼
Country average significantly higher than international average		▲
No statistically significant difference between country average and international average		●
Country average significantly lower than international average		▼
Significance tests adjusted for multiple comparisons		

SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1998-1999.

* The item was answered correctly by a majority of students reaching this benchmark.

† Met guidelines for sample participation rates only after replacement schools were included (see Exhibit A.8).

1 National Desired Population does not cover all of International Desired Population (see Exhibit A.5). Because coverage falls below 65%, Latvia is annotated LSS for Latvian-Speaking Schools only.

2 National Defined Population covers less than 90 percent of National Desired Population (see Exhibit A.5).

‡ Lithuania tested the same cohort of students as other countries, but later in 1999, at the beginning of the next school year.

Internationally comparable data are unavailable for Morocco, Thailand, and Tunisia.

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

What Issues Emerge from the Benchmark Descriptions?

The benchmark descriptions and example items strongly suggest a gradation in achievement, from the top-performing students' ability to generalize and solve non-routine or contextualized problems to the lower-performing students being able primarily to use routine, mainly numeric procedures. The fact that even at the Median Benchmark students demonstrate only limited achievement in problem solving beyond straightforward one-step problems may suggest a need to reconsider the role, or priority, of problem solving in mathematics curricula.

In looking across the item-level results, it also is important to note the variation in performance across the topics covered. For example, on just the few items (16) presented in this chapter, there was a substantial range in performance for many countries. While some countries consistently registered high or low performance, and others had results consistently near the international average, 16 countries performed significantly above the international average on at least one item, and significantly below the international average on at least one item (Australia, Bulgaria, Canada, Cyprus, England, Finland, Latvia (LSS), Lithuania, Malaysia, Moldova, the Netherlands, New Zealand, Romania, Thailand, Tunisia, and the United States). For example, Malaysia had the highest percent correct on a subtraction item (Exhibit 2.19) but performed below the international average on an item requiring selection of information to solve a complex word problem (Exhibit 2.3). In some cases, differences of this sort will result from intended differences in emphasis in national curricula. It is likely, however, that such results may be unintended, and the findings will provide important information about strengths and weaknesses in intended or implemented curricula. At the very least, an in-depth examination of the TIMSS 1999 results may reveal aspects of curricula that merit further investigation.